

Robust planning of dynamic wireless charging infrastructure for battery electric buses

Zhaocai Liu, Ziqi Song*

Department of Civil and Environmental Engineering, Utah State University, Logan, UT
84322-4110, United States

* Corresponding author.
E-mail: ziqi.song@usu.edu

Abstract

Battery electric buses with zero tailpipe emissions have great potential in improving environmental sustainability and livability of urban areas. However, the problems of high cost and limited range associated with on-board batteries have substantially limited the popularity of battery electric buses. The technology of dynamic wireless power transfer (DWPT), which provides bus operators with the ability to charge buses while in motion, may be able to effectively alleviate the drawbacks of electric buses. In this paper, we address the problem of simultaneously selecting the optimal location of the DWPT facilities and designing the optimal battery sizes of electric buses for a DWPT electric bus system. The problem is first constructed as a deterministic model in which the uncertainty of energy consumption and travel time of electric buses is neglected. The methodology of robust optimization (RO) is then adopted to address the uncertainty of energy consumption and travel time. The affinely adjustable robust counterpart (AARC) of the deterministic model is developed, and its equivalent tractable mathematical programming is derived. Both the deterministic model and the robust model are demonstrated with a real-world bus system. The results demonstrate that the proposed deterministic model can effectively determine the allocation of DWPT facilities and the battery sizes of electric buses for a DWPT electric bus system; and the robust model can further provide optimal designs that are robust against the uncertainty of energy consumption and travel time for electric buses.

Keywords:

Battery electric bus; Dynamic wireless power transfer; Wireless charging, System optimization; Robust optimization; Affinely adjustable robust counterpart.

1. Introduction

As an integral part of public transportation, the public bus system provides people with an economical and sustainable travel mode, and it helps to reduce traffic congestion and exhaust emissions. However, due to the limitations of vehicle technology, diesel-powered buses still dominate today's bus fleet. For example, diesel buses accounted for 50.5% of all bus vehicles in the United States in 2015 (Dickens and Neff, 2016). Diesel engines are a primary source of particulate matter (PM) and nitrogen oxides (NO_x) emitted by motor vehicles. Furthermore, most transit buses are operated in densely populated urban areas, and they are generally in use for large portions of the day. Battery electric buses, which produce zero tailpipe emissions, offer tremendous potential in improving the environmental sustainability and livability of urban areas. However, range limitations associated with on-board batteries as well as the problem of battery size, cost, and life, have substantially limited the popularity of electric buses.

The technology of dynamic wireless power transfer (DWPT), also called dynamic inductive charging, offers the promise of eliminating the range limitation of electric buses. DWPT provides bus operators the ability to charge buses while in motion, using wireless inductive power transfer pads embedded underneath the roadway. The technology potentially makes electric buses as capable as their diesel counterparts. DWPT technology has been implemented in a bus line in Gumi City, South Korea (Jang et al., 2015). Utah State University (USU) demonstrated the DWPT technology for an electric bus with peak power of 25 kW at its Electric Vehicle and Roadway (EVR) test track in 2016 (see Fig. 1). Additionally, the United Kingdom recently conducted a study to determine the feasibility of implementing this technology on its strategic road network (Highways England, 2015). Another benefit of DWPT technology is that it could substantially reduce on-board battery size. The battery pack on a long-range all-electric bus can account for about a quarter of the weight of the vehicle and as much as 39% of the total cost of the bus (Bi et al., 2015). Bi et al. (2015) demonstrated the potential of downsizing the battery of an electric bus to about one-third of a plug-in charged battery, assuming stationary wireless charging at bus stations is employed. The battery downsizing not only makes electric buses more affordable, but also offers additional energy savings, due to reduced vehicle weight.

“place Fig. 1 about here”

Although a number of studies have investigated the problem of deploying or managing DWPT facilities for private electric vehicles in transportation networks (e.g., He et al., 2013; Riemann, 2015; Chen et al., 2016, 2017; Fuller, 2016; Deflorio and Castello, 2017), with current technologies, constructing DWPT facilities for private electric vehicles could be costly. Fuller (2016) estimated that it costs \$4 million per lane mile to construct DWPT facilities for private electric vehicles. However, constructing DWPT facilities for an electric bus system is quite different from constructing such facilities for private electric vehicles. DWPT facilities consist of inverters and wireless power transfer pads. For DWPT facilities for private electric vehicles, inverters should be densely deployed to serve continuous vehicle flows. However, headways of buses can be controlled through proper scheduling. As a result, for DWPT facilities for an electric bus system, an inverter can cover a relatively long distance of roadway. Therefore, the cost for constructing DWPT facilities for an electric bus system could be significantly reduced.

To enable DWPT for an electric bus system, wireless charging infrastructure must be strategically built in the road network. Meanwhile, because DWPT provides the potential of reducing on-board battery size, battery sizes for electric buses should also be designed. The charging infrastructure planning problem is twofold. First, the combination of deployed dynamic wireless charging facilities and designed battery sizes should ensure the normal operation of electric buses. Second, one must consider the trade-off between on-board battery sizes and the number (length) of DWPT facilities.

A handful of studies have investigated the location of DWPT infrastructure for electric buses. Ko and Jang (2011) formulated a nonlinear model to simultaneously determine the optimal location of DWPT facilities and the battery sizes of electric buses for a single electric bus line. In this model, the cost of DWPT facilities is linearly related to length. Ko and Jang (2013) improved this model by separating the cost of DWPT facilities into two parts: the cost of inverters and the cost of cables. The total number of DWPT facilities determines the cost of inverters, and the cost of cables is linearly related to the total length. More

recently, [Jang et al. \(2015\)](#) proposed a mixed-integer programming (MIP) model to optimize the location of DWPT facilities and the battery sizes of electric buses for a DWPT electric bus line in a closed environment.

The above studies only consider electric bus systems with a single bus line. However, a real-world bus system almost always contains more than one bus line. Moreover, multiple transit lines may have significant overlap, especially in areas with high transit demand, e.g., downtown or shopping malls. Overlapping transit lines could share wireless power transfer pads. The synergistic effect among different transit lines could substantially reduce the average cost of constructing DWPT infrastructure for individual bus lines and make DWPT more economically attractive for real-world implementation. Another significant drawback of previous studies lies in their strong assumption that energy consumption and travel time of electric buses are predefined. Nevertheless, in real-world traffic, energy consumption and travel time of electric buses will change along with traffic conditions and travel demands. Note that the travel time of an electric bus on a DWPT facility determines the potential dynamic charging time. Ignoring the uncertainty of energy consumption and travel time of electric buses could lead to a suboptimal or even infeasible plan for a DWPT electric bus system.

In this paper, we consider the planning problem of DWPT infrastructure in a general electric bus system with multiple lines. Moreover, the uncertainty of energy consumption and travel time of electric buses is also considered through robust optimization (RO). The primary contributions of our work are summarized as follows:

- We develop an innovative model to select the optimal location of DWPT facilities and design the optimal battery sizes of electric buses for a DWPT electric bus system with multiple lines.
- Based on the deterministic model, we formulate the corresponding robust optimization model, which can provide robust optimal solutions against the uncertainty of energy consumption and travel time of electric buses.
- We reformulate the initial robust optimization model, which is intractable, into a computationally tractable model.

The remaining portions of this paper are organized as follows. In the following section, we formulate a deterministic model to optimize the location of DWPT facilities and the battery sizes of electric buses for a DWPT electric bus system. Next, in Section 3 we propose a robust counterpart model to consider the uncertainty of energy consumption and travel time of electric buses. Section 4 presents numerical studies for both deterministic model and robust model. Finally, conclusions are discussed in Section 5.

2. Deterministic optimization model

In this section, we first introduce the optimization issue of a DWPT electric bus system, and we then provide the network representation of a DWPT electric bus system. Next, we present the decision variables and constraints of our model. Finally, we formulate an optimization model to select the optimal locations of DWPT facilities and design optimal battery sizes of electric buses for a DWPT electric bus system. Note that all input parameters in the model are predefined in this section. Thus, this model is deterministic.

2.1 The optimization issue of a DWPT electric bus system

A DWPT electric bus system consists of DWPT facilities and electric buses. As shown in Fig. 2, an independent DWPT facility consists of an inverter and a series of wireless power transfer pads that are installed underneath the road. Electric buses can be charged while moving over these pads. Compared with traditional electric buses, which can only be charged when idle, electric buses in a DWPT electric bus system could carry smaller batteries because they can be charged en route. Through the implementation of DWPT for an electric bus system, the cost of on-board batteries is reduced. If we want to save more money on batteries for electric buses, we need to install more DWPT facilities at appropriate locations, which means that additional investments will be required for DWPT facilities. Thus, for a DWPT electric bus

system to be effective, there must be an optimal trade-off between the cost of batteries and the cost of DWPT infrastructure.

“place Fig. 2 about here”

To optimize a DWPT electric bus system, we need to simultaneously determine the battery sizes of electric buses and the allocation of DWPT facilities. The combination of the battery sizes of electric buses and the allocation of DWPT facilities should first meet the energy requirement for normal operations of the electric bus system. Based on this requirement, we can then minimize the total cost of batteries and DWPT facilities. Furthermore, our deterministic optimization model is based on a DWPT electric bus system operating under the following assumptions:

- 1) Each bus line in the bus system operates on a fixed route.
- 2) Each bus line has a base station, where all buses start and end each of their service loops.
- 3) Once an electric bus completes a service loop, it will be fully charged at the base station before it starts another service loop.
- 4) The speed profile and the number of boarding/alighting passengers at bus stations are predefined.

The above assumptions are introduced for system modeling purposes, and they are not restrictive. Assumptions 1) and 2) are common, even for traditional bus systems. Assumption 3) requires electric buses to stay at the base station for a certain period of time and be fully charged after completing each service loop. Assumption 4) ensures that the input parameters of our model (i.e., energy consumption and travel time of electric buses) are deterministic. Based on the preference of the decision maker, the speed profile and the number of boarding/alighting passengers at bus stations could be the expected value or the worst-case value. Furthermore, assumption 4) was also adopted by previous studies on the DWPT electric bus system. (e.g., Ko and Jang, 2011; Ko and Jang, 2013; Jang et al., 2015).

2.2 Network representation of a DWPT electric bus system

Let $G(N, L)$ denote the road network of the electric bus system, where N is the set of nodes and L is the set of directed links. A bidirectional road is treated as two unidirectional roads. To locate the DWPT facilities accurately in the network, we further divide each road segment into a set of short links. The location problem of charging facilities is then converted into determining whether to install charging facilities on certain links. Consider a DWPT electric bus system that includes several bus lines. An independent DWPT facility is located on a series of adjacent links. Let K denote the set of all electric bus lines. For the convenience of modeling, the base station of a bus line $k \in K$ is represented by two nodes O_k^s and O_k^e , which denote the starting point and the ending point of a service loop, respectively. L is represented as node pairs (i, j) , where $i, j \in N$ and $i \neq j$. Let d_{ij} denote the length of link (i, j) . Let L_k denote the set of all of the links that form the route of bus line k and let N_k denote the corresponding nodes, where L_k and N_k are subsets of L and N , respectively.

2.3 Decision variables

Our model has two groups of decision variables that determine the location of DWPT facilities and the battery sizes of electric buses, respectively. Table 1 shows a summary of the variables introduced to represent the location of DWPT facilities and to count the total number of independent DWPT facilities. Table 2 shows a summary of the variables introduced to represent the battery sizes and battery levels of electric buses. The specific definitions of these variables are introduced in the next section.

“place Table 1 about here”

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2.4 Constraints

In this section, we introduce the constraints in our model. Our model has constraints on DWPT facilities as well as constraints on energy requirements.

2.4.1 Constraints on DWPT facilities

As introduced above, each independent DWPT facility consists of one inverter and a series of wireless power transfer pads. These power transfer pads are installed on a set of adjacent links, and they share one inverter. To locate DWPT facilities in the network, we introduce a binary variable x_{ij} for each link (i, j) to represent whether it is covered by a DWPT facility.

$$x_{ij} = \begin{cases} 1 & \text{if link } (i, j) \text{ is covered by a DWPT facility} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The cost of a DWPT facility consists of the cost of an inverter and the cost of wireless power transfer pads. The cost of an inverter is a fixed cost, because a DWPT facility needs an inverter regardless of the length of the wireless power transfer pads. The cost of wireless power transfer pads should be a variable cost depending on the length. In this paper, we assume that the cost of wireless power transfer pads is proportional to the length of the power transfer pads. To evaluate the total cost of DWPT facilities, we must determine the number of inverters and the total length of wireless power transfer pads. Based on the definition of binary variable x_{ij} , the total length of power transfer pads can be readily given by $\sum_{(i,j) \in L} (x_{ij} d_{ij})$. As for the number of inverters (i.e., the number of independent DWPT facilities), we must introduce new variables to determine its value.

Because links in the network have directions, we can define the concept of starting points for DWPT facilities as follows:

Definition 1: For a node i , if it has no incoming links or all of its incoming links $(m, i) \in L$ are not covered by DWPT facilities, and it has one or more outgoing links (i, j) covered by a DWPT facility, we define node i as a starting point of the DWPT facility.

As shown in Fig. 3, a DWPT facility is built on a road segment represented by three links. Based on our definition, the node i is a starting point of the DWPT facility.

“place Fig. 3 about here”

We introduce a binary variable y_i to denote whether node i is a starting point of a DWPT facility.

$$y_i = \begin{cases} 1 & \text{if node } i \text{ is a starting point of a DWPT facility} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Based on the definition of binary variables x_{ij} and y_i , we obtain the following conditional constraints.

$$y_i \leq \sum_{(i,j) \in L_i^+} x_{ij}, \quad \forall i \in N \quad (3)$$

$$y_i \leq 1 - x_{mi}, \quad \forall i \in N, \forall (m, i) \in L_i^- \quad (4)$$

$$y_i \geq x_{ij} - \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N, \forall (i, j) \in L_i^+ \quad (5)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in L \quad (6)$$

$$y_i \in \{0, 1\} \quad \forall i \in N \quad (7)$$

where L_i^+ and L_i^- are the set of outgoing and incoming links for node $i \in N$, respectively.

For a node $i \in N$, constraint (3) ensures that if it has no outgoing links, or all of its outgoing links (i, j) are not covered by DWPT facilities, it cannot be a starting point for a DWPT facility. Constraint (4) requires that if a node i has an incoming link (m, i) covered by a DWPT facility, it cannot be a starting point for a DWPT facility. Constraint (5) ensures that if a node i has no incoming links, or all of its incoming links $(m, i) \in L$ are not covered by DWPT facilities, and it has one or more outgoing links (i, j) covered by a DWPT facility, node i must be a starting point of the DWPT facility.

The total number of independent DWPT facilities can be easily given by $\sum_{i \in N} y_i$ if each DWPT facility has one and only one starting point. However, there are two cases in which a DWPT facility does not correspond to a starting point. First is that when a DWPT facility is built on a set of links that form a head-to-tail cycle, it will have no starting point. Though we can detect potential cycles in the network beforehand and arbitrarily assign one node in a cycle to be the starting point, for simplicity, we assume in this paper that the network contains no directed cycles. Second, as shown in Fig. 4, when a DWPT facility is

branch-like and covers two or more road segments (each of which is represented by a set of links) that merge at the same intersection, it will have more than one starting point. Therefore, $\sum_{i \in N} y_i$ that represents the total number of starting points of DWPT facilities needs to be revised to get the accurate number of DWPT facilities.

“place Fig. 4 about here”

Let N^s denote the set of all intersection nodes. For each node $i \in N^s$, we introduce a binary variable z_i to represent whether node i has incoming links covered by a DWPT facility.

$$z_i = \begin{cases} 1 & \text{if node } i \text{ has incoming links covered by a DWPT facility} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

This statement can be represented by the following constraints.

$$z_i \leq \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N^s \quad (9)$$

$$z_i \geq x_{mi} \quad \forall i \in N^s, \forall (m,i) \in L_i^- \quad (10)$$

Constraint (9) ensures that if all incoming links of the node $i \in N^s$ are not covered by any DWPT facilities, z_i will be zero. Constraint (10) ensures that if a node $i \in N^s$ has one or more incoming links (m,i) covered by a DWPT facility, z_i will be one.

For a node $i \in N^s$, each of its incoming links (m,i) , if covered by a DWPT facility, can trace back to a starting point of the DWPT facility. Thus, we can use $\sum_{(m,i) \in L_i^-} x_{mi}$ to count the number of starting points directing node i . In addition, binary variable z_i indicates whether a node $i \in N^s$ has one or more incoming links covered by a DWPT facility. Through subtracting $\sum_{(m,i) \in L_i^-} x_{mi} - z_i$ for each node $i \in N^s$ from the total number of starting points $\sum_{i \in N} y_i$, we can obtain the accurate total number of DWPT facilities as follows:

$$\sum_{i \in N} y_i - \sum_{i \in N^s} \left(\sum_{(m,i) \in L_i^-} x_{mi} - z_i \right)$$

2.4.2 Constraints on energy requirement

For a DWPT electric bus system, its deployed DWPT facilities and equipped battery sizes should satisfy the energy requirement for normal operations. Based on our network representation, the service route of each electric bus line consists of a series of links. When an electric bus travels on these links, its battery level will change due to the energy consumption and energy supply (i.e., possible charging from DWPT facilities). For an electric bus of bus line $k \in K$, let e_{ki} denote its battery level at node $i \in N_k$, and when it traverses a link $(i,j) \in L_k$, let c_{kij} and s_{kij} denote the energy consumption and energy supply on link (i,j) , respectively. Then we have the following battery level recurrence equation:

$$e_{kj} = e_{ki} - c_{kij} + s_{kij} \quad (i,j) \in L_k \quad (11)$$

To preserve battery life, the battery level of an electric bus should be within the range of lower and upper limits.

$$e_{ki} \leq e_k^{up} \quad \forall k \in K, \forall i \in N_k \quad (12)$$

$$e_{ki} \geq e_k^{lo} \quad \forall k \in K, \forall i \in N_k \quad (13)$$

where e_k^{lo} and e_k^{up} are the lower and upper limits of the battery level for electric buses on line k , respectively. The e_k^{lo} and e_k^{up} are usually set by battery providers. Use beyond the range between e_k^{lo} and e_k^{up} will damage the battery and thus shorten the battery life. Let e_k^{max} denote the battery size of the electric buses on line $k \in K$. Usually e_k^{lo} and e_k^{up} are given by the following equations:

$$e_k^{up} = \epsilon_k^{up} e_k^{max} \quad \forall k \in K \quad (14)$$

$$e_k^{lo} = \epsilon_k^{lo} e_k^{max} \quad \forall k \in K \quad (15)$$

where ϵ_k^{lo} and ϵ_k^{up} are the predetermined coefficients, and $0 < \epsilon_k^{lo} < \epsilon_k^{up} < 1$.

Substituting Eq. (14) into constraint (12) and substituting Eq. (15) into constraint (13) yield the following battery level constraints

$$e_{ki} \leq \epsilon_k^{up} e_k^{max} \quad \forall k \in K, \forall i \in N_k \quad (16)$$

$$e_{ki} \geq \epsilon_k^{lo} e_k^{max} \quad \forall k \in K, \forall i \in N_k \quad (17)$$

Additionally, decision variable e_k^{max} should satisfy the following non-negativity constraints:

$$e_k^{max} \geq 0 \quad \forall k \in K \quad (18)$$

In a DWPT electric bus system, each electric bus is assumed to be fully charged when it starts from its base station (i.e., see assumption 3). Thus, we have:

$$e_{ki} = e_k^{up} = \epsilon_k^{up} e_k^{max} \quad \forall k \in K, \forall i = O_k^s \quad (19)$$

For an electric bus on line $k \in K$, using Eq. (11) and Eq. (19), we can obtain its battery level at any node $i \in N_k$.

To evaluate the energy consumption and energy supply on each link, we propose the following two models.

Energy consumption model

The energy consumption of an electric bus depends on many factors, such as velocity, mass, road gradient and use of accessory devices. A comprehensive formulation of c_{kij} is given as the following function:

$$c_{kij} = F(v_{kij}, \dot{v}_{kij}, \theta_{ij}, w_{kij})$$

where v_{kij} and \dot{v}_{kij} are the average velocity and acceleration of an electric bus on line k within the range of link (i, j) , respectively. θ_{ij} refers to the average grade of link (i, j) . w_{kij} represents the total mass of an electric bus on line k when it travels on link (i, j) . Based on the energy consumption model proposed by Wang et al. (2013), we present our energy consumption model of an electric bus as follows:

$$\begin{aligned} c_{kij} = & \left\{ \eta_k^{out} \left(\varpi_{ij} w_{kij} \varepsilon + \frac{\rho}{2} \sigma \Gamma_k (v_{kij})^2 \right) \right. \\ & + \left(\eta_k^{out} \xi_{ij}^\theta + \eta_k^{in} (1 - \xi_{ij}^\theta) \right) w_{kij} \varepsilon \sin \theta_{ij} \\ & \left. + d_{ij} \left(\eta_k^{out} \xi_{kij}^{\dot{v}} + \eta_k^{in} (1 - \xi_{kij}^{\dot{v}}) \right) w_{kij} \dot{v}_{kij} \right\} \quad \forall k \in K, \forall (i, j) \in L_k \end{aligned} \quad (20)$$

where ϖ_{ij} is the rolling friction coefficient on link (i, j) , ε represents the gravity acceleration, ρ is the air density, σ is the coefficient of air resistance, and Γ_k represents the frontal area of an electric bus. η_k^{out} and η_k^{in} are the energy output and input efficiency of an electric bus on line k , respectively, and $\eta_k^{out} > 1 > \eta_k^{in}$. Note that for simplicity, the energy consumption of auxiliary electric devices, such as air conditioners and lights, is not considered in our model, although it can be evaluated through the product of travel time and the power of corresponding devices. ξ is defined as follows:

$$\begin{aligned} \xi_{ij}^\theta &= \begin{cases} 1, & \theta_{ij} > 0 \\ 0, & \theta_{ij} \leq 0 \end{cases} \quad \forall (i, j) \in L_k \\ \xi_{kij}^{\dot{v}} &= \begin{cases} 1, & \dot{v}_{kij} > 0 \\ 0, & \dot{v}_{kij} \leq 0 \end{cases} \quad \forall k \in K, \forall (i, j) \in L_k \end{aligned}$$

As mentioned in the introduction, the battery pack on a long-range all-electric bus can account for a significant portion of the weight of the vehicle. With a smaller battery pack, the energy consumption of an electric bus will also be reduced. To further consider the impact of the weight of the battery pack on the energy consumption, we divide the total weight of an electric bus into a fixed part and a variable part, where the variable part represents the weight of the battery pack. Let w_{kij}^{fix} denote the fixed part of the weight of an electric bus and let w_{kij}^{bat} denote the weight of the battery pack. The battery used in electric buses is a pack of multiple battery cells, and the amount of the battery cells determines the energy capacity and the weight. Therefore, without loss of generality, we assume that w_{kij}^{bat} is given by the following equation:

$$w_{kij}^{bat} = b e_k^{max}$$

where b is a parameter representing the weight of battery pack per unit capacity. If we replace the w_{kij} in equation (20) with $w_{kij}^{fix} + w_{kij}^{bat}$, the energy consumption c_{kij} becomes the following linear function of e_k^{max} :

$$c_{kij} = \left\{ \eta_k^{out} \left(\varpi_{ij} w_{kij}^{fix} \varepsilon + \frac{\rho}{2} \sigma \Gamma_k (v_{kij})^2 \right) + \left(\eta_k^{out} \xi_{ij}^\theta + \eta_k^{in} (1 - \xi_{ij}^\theta) \right) w_{kij}^{fix} \varepsilon \sin \theta_{ij} + d_{ij} \left(\eta_k^{out} \xi_{kij}^{\dot{v}} + \eta_k^{in} (1 - \xi_{kij}^{\dot{v}}) \right) w_{kij}^{fix} \dot{v}_{kij} \right\} + \left\{ \eta_k^{out} \varpi_{ij} \varepsilon + \left(\eta_k^{out} \xi_{ij}^\theta + \eta_k^{in} (1 - \xi_{ij}^\theta) \right) \varepsilon \sin \theta_{ij} + d_{ij} \left(\eta_k^{out} \xi_{kij}^{\dot{v}} + \eta_k^{in} (1 - \xi_{kij}^{\dot{v}}) \right) \dot{v}_{kij} \right\} b e_k^{max} \quad (21)$$

In this model, the parameters v_{kij} , \dot{v}_{kij} , θ_{ij} and w_{kij}^{fix} are all predefined input data. θ_{ij} is determined by the geological condition of the road network, which can be obtained from GIS data or through field measurements. Based on assumption 4) that the speed profile and the number of boarding/alighting passengers are predefined, v_{kij} , \dot{v}_{kij} and w_{kij}^{fix} are all deterministic parameters. Thus, energy consumption c_{kij} can be represented as the following simplified form:

$$c_{kij} = c_{kij}^{fix} + c_{kij}^{unit} e_k^{max} \quad (22)$$

where $c_{kij}^{fix} = \left\{ \eta_k^{out} \left(\varpi_{ij} w_{kij}^{fix} \varepsilon + \frac{\rho}{2} \sigma \Gamma_k (v_{kij})^2 \right) + \left(\eta_k^{out} \xi_{ij}^\theta + \eta_k^{in} (1 - \xi_{ij}^\theta) \right) w_{kij}^{fix} \varepsilon \sin \theta_{ij} + d_{ij} \left(\eta_k^{out} \xi_{kij}^{\dot{v}} + \eta_k^{in} (1 - \xi_{kij}^{\dot{v}}) \right) w_{kij}^{fix} \dot{v}_{kij} \right\}$ and $c_{kij}^{unit} = \left\{ \eta_k^{out} \varpi_{ij} \varepsilon + \left(\eta_k^{out} \xi_{ij}^\theta + \eta_k^{in} (1 - \xi_{ij}^\theta) \right) \varepsilon \sin \theta_{ij} + d_{ij} \left(\eta_k^{out} \xi_{kij}^{\dot{v}} + \eta_k^{in} (1 - \xi_{kij}^{\dot{v}}) \right) \dot{v}_{kij} \right\} b$. c_{kij}^{fix} and c_{kij}^{unit} are predetermined parameters. c_{kij}^{fix} represents the fixed part of energy consumption while $c_{kij}^{unit} e_k^{max}$ represents the energy consumption caused by the weight of the battery pack.

Note that this model excludes the possible energy input from DWPT facilities. The energy supply model is given below.

Energy supply model

If a link $(i, j) \in L_k$ is covered by a DWPT facility, the energy supply s_{kij} will be determined by the charging rate and the actual charging time. For simplicity, in our model, we assume the charging rate of DWPT facilities is constant. Let p denote the charging rate, which is also the energy supply rate. Let t_{kij} denote the travel time of an electric bus of line k on link $(i, j) \in L_k$. Every link in the bus network is a candidate location for DWPT facilities. Since binary variable x_{ij} represents whether link (i, j) is covered by a DWPT facility, the maximum potential energy supply on link $(i, j) \in L_k$ for an electric bus on line k can be given by $x_{ij} p t_{kij}$. Moreover, assume that bus drivers can decide whether to charge when electric buses are moving on a DWPT facility, the actual energy supply s_{kij} then should satisfy the following constraints:

$$s_{kij} \leq x_{ij} p t_{kij} \quad \forall k \in K, \forall (i, j) \in L_k \quad (23)$$

$$s_{kij} \geq 0 \quad \forall k \in K, \forall (i, j) \in L_k \quad (24)$$

Substituting Eq. (11) into constraints (23) and (24) and replacing c_{kij} with equation (22) provides two additional battery level constraints as follows:

$$e_{kj} \leq e_{ki} - c_{kij}^{fix} - c_{kij}^{unit} e_k^{max} + x_{ij} p t_{kij} \quad \forall k \in K, \forall (i, j) \in L_k \quad (25)$$

$$e_{kj} \geq e_{ki} - c_{kij}^{fix} - c_{kij}^{unit} e_k^{max} \quad \forall k \in K, \forall (i, j) \in L_k \quad (26)$$

Note that in our model, the degeneration of batteries is not considered. We assume that the battery capacity of an electric bus will not change during its service life. In the system level design, ignoring the change of battery capacity is a common practice (Jang et al., 2015; Ashtari et al., 2012; Mohrehkesh and Nadeem, 2011).

2.5 System optimization model of a DWPT electric bus system

The objective function of our model is the total cost of batteries and DWPT facilities. Based on the service life of batteries and DWPT facilities, we amortize the cost over the lifespan of a DWPT electric bus system. As a result, all of the costs mentioned below are the amortized costs. The cost of DWPT facilities includes the fixed cost and the variable cost as aforementioned. The fixed cost of a DWPT facility is denoted as a^{fix} , and the variable cost per unit length is denoted as a^{var} . In Section 2.4.1, we have obtained the total length and total number of DWPT facilities. Thus, the total cost of DWPT facilities can be given by the following:

$$a^{fix} \left\{ \sum_{i \in N} y_i - \left(\sum_{i \in N^s} \sum_{(m,i) \in L_i^-} x_{mi} - \sum_{i \in N^s} z_i \right) \right\} + a^{var} \sum_{(i,j) \in L} d_{ij} x_{ij}$$

The cost of a battery depends on its capacity. The battery used in electric buses is a pack of multiple battery cells, and the amount of the battery cells determines the energy capacity and the cost. Here, we adopt the widely used approximation that the battery cost is linearly proportional to the battery capacity (Li, 2013). The battery cost per unit capacity is denoted as a^{bat} . Let ζ_k denote the number of electric buses that operate on line k . This parameter is predetermined. For a DWPT electric bus system, the total cost of the batteries is as follows:

$$a^{bat} \sum_{k \in K} \zeta_k e_k^{max}$$

Based on all of the discussions above, we develop a system optimization model (S1) for a DWPT electric bus system as follows. For completeness, we repeat some previously presented constrains here.

$$(S1): \min_{(x_{ij}, y_i, z_i, e_{ki}, e_k^{max})} a^{fix} \left\{ \sum_{i \in N} y_i - \left(\sum_{i \in N^s} \sum_{(m,i) \in L_i^-} x_{mi} - \sum_{i \in N^s} z_i \right) \right\} + a^{var} \sum_{(i,j) \in L} d_{ij} x_{ij} \\ + a^{bat} \sum_{k \in K} \zeta_k e_k^{max}$$

s. t.

$$y_i \leq \sum_{(i,j) \in L_i^+} x_{ij}, \quad \forall i \in N \quad (27)$$

$$y_i \leq 1 - x_{mi}, \quad \forall i \in N, \forall (m,i) \in L_i^- \quad (28)$$

$$y_i \geq x_{ij} - \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N, \forall (i,j) \in L_i^+ \quad (29)$$

$$z_i \leq \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N^s \quad (30)$$

$$z_i \geq x_{mi} \quad \forall i \in N^s, \forall (m,i) \in L_i^- \quad (31)$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in L \quad (32)$$

$$y_i \in \{0,1\} \quad \forall i \in N \quad (33)$$

$$z_i \in \{0,1\} \quad \forall i \in N^s \quad (34)$$

$$e_{ki} = \epsilon_k^{up} e_k^{max} \quad \forall k \in K, \forall i = O_k^s \quad (35)$$

$$e_{kj} \leq e_{ki} - c_{kij}^{fix} - c_{kij}^{unit} e_k^{max} + pt_{kij} x_{ij} \quad \forall k \in K, \forall (i,j) \in L_k \quad (36)$$

$$e_{kj} \geq e_{ki} - c_{kij}^{fix} - c_{kij}^{unit} e_k^{max} \quad \forall k \in K, \forall (i,j) \in L_k \quad (37)$$

$$e_{ki} \leq \epsilon_k^{up} e_k^{max} \quad \forall k \in K, \forall i \in N_k \quad (38)$$

$$e_{ki} \geq \epsilon_k^{lo} e_k^{max} \quad \forall k \in K, \forall i \in N_k \quad (39)$$

$$e_k^{max} > 0 \quad \forall k \in K \quad (40)$$

3. Robust formulation

3.1 Robust optimization

Although the proposed deterministic model can solve the optimal design problem of a DWPT electric bus system, the solution to an optimization problem could be very sensitive to perturbations in the parameters of the problem. Without considering the parameters' uncertainty, the optimal solutions could turn out to be infeasible and suboptimal (Bertsimas et al., 2011). In the domain of planning, much attention has been given to data uncertainty in past years, and various modeling techniques are used to address the uncertainty of input data and parameters. The main approaches consist of two groups: stochastic programming (SP) and robust optimization (RO). The SP approach assumes the uncertain data to be random and requires known probability distribution. Moreover, the commonly used chance-constrained programming in SP is rarely computationally tractable.

On the other hand, the RO approach includes scenario-based RO and set-based RO. The scenario-based RO approach for general linear programming (LP) problems was first proposed by Mulvey et al. (1995). This approach has been used in the network design problem (NDP) (see Karoonsoontawong and Waller, 2007; Ukkusuri et al., 2007) and traffic signal timing (see Yin, 2008). The scenario-based RO approach also requires the probability of each scenario, and the computational work could be very expensive when the number of scenarios is large. In the set-based RO approach, the uncertain parameters are considered in a given set, and the solutions need to be feasible for any realization of the uncertainty in the set. Thus, the set-based RO model is not stochastic but rather deterministic. The theoretical framework of the set-based RO approach has been developed and improved by many researchers (e.g., Ben-Tal and Nemirovski, 1998; Ben-Tal and Nemirovski, 1999; El Ghaoui and Lebret, 1997; El Ghaoui et al., 1998). The application of the set-based RO approach has also been identified in many study areas. Ben-Tal et al. (2011) considered the demand uncertainty in humanitarian relief supply chains and proposed a methodology to provide a robust logistics plan. A polyhedral uncertainty set, which is the intersection of the box uncertainty set and the budget uncertainty set, is used to bind demand uncertainty. Additionally, the affinely adjustable robust counterpart (AARC) approach is adopted to consider "wait and see" decisions and to provide less-conservative solutions. Lu (2013) developed a robust multi-period fleet allocation model for bike-sharing systems by considering the time-dependent demand with convex hull and ellipsoidal uncertainty sets. Chung et al. (2011) applied the RO approach in the dynamic network design problem and used a box uncertainty set to characterize demand uncertainty. Evers et al. (2014) considered uncertain fuel consumption in the mission planning problem of unmanned aerial vehicles (UAVs). Different uncertainty sets, including box uncertainty set, budget uncertainty set, ellipsoid uncertainty set and their intersections, are adopted to describe the fuel consumption uncertainty.

In this section, based on our newly developed model regarding the optimal design of a DWPT electric bus system, we further propose the robust counterpart model. The uncertainty of energy consumption and energy supply of electric buses is explicitly considered. The approach developed by Ben-Tal et al. (2009) is used to derive the robust counterpart for a given uncertainty set.

3.2 Uncertainty set

In the robust model, the fixed part of energy consumption c_{kij}^{fix} and maximum possible charging time t_{kij} are no longer deterministic. Instead, they are given by an uncertainty set. Let \bar{c}_{kij}^{fix} and \bar{t}_{kij} denote the expected value of c_{kij}^{fix} and t_{kij} , respectively, and let \hat{c}_{kij}^{fix} and \hat{t}_{kij} denote the maximum deviation of c_{kij}^{fix} and t_{kij} , respectively. The actual realizations of c_{kij}^{fix} and t_{kij} can then be expressed by the following:

$$\begin{aligned} c_{kij}^{fix} &= \bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix} \\ t_{kij} &= \bar{t}_{kij} + \psi_{kij} \hat{t}_{kij} \end{aligned}$$

where $\varphi_{kij}, \psi_{kij} \in [-1, 1]$. The uncertainty of c_{kij}^{fix} and t_{kij} can be then represented by variable realizations of φ_{kij} and ψ_{kij} . The commonly used box uncertainty set is given as follows:

$$\Phi_k \in U_k^{box1} = \{ \Phi_k \in \mathbb{R}^{|L_k|} \mid \|\Phi_k\|_\infty \leq 1 \} = \left\{ \Phi_k \in \mathbb{R}^{|L_k|} \mid \max_{(i,j) \in L_k} |\varphi_{kij}| \leq 1 \right\}$$

$$\Psi_k \in U_k^{box2} = \{\Psi_k \in \mathbb{R}^{|L_k|} \mid \|\Psi_k\|_\infty \leq 1\} = \left\{ \Psi_k \in \mathbb{R}^{|L_k|} \mid \max_{(i,j) \in L_k} |\psi_{kij}| \leq 1 \right\}$$

where Φ_k is a vector of $\{\dots, \varphi_{kij}, \dots\}$ and Ψ_k is a vector of $\{\dots, \psi_{kij}, \dots\}$, $(i, j) \in L_k$. $|L_k|$ represents the total number of links in set L_k . $\|\Phi_k\|_\infty = \max_{(i,j) \in L_k} |\varphi_{kij}|$ and $\|\Psi_k\|_\infty = \max_{(i,j) \in L_k} |\psi_{kij}|$ are the maximum norms of Φ_k and Ψ_k , respectively. Note that for simplicity, here we assume that Φ_k and Ψ_k belong to two independent uncertainty sets, even though we can add additional constraints for the uncertainty set to consider the correlation between them. Additionally, uncertainty sets for different bus lines are also assumed to be independent. The uncertainty level of the box uncertainty set can be represented by the ratio of maximum deviation and the expected value (i.e. $\hat{c}_{kij}^{fix}/\bar{c}_{kij}^{fix}$ and $\hat{t}_{kij}/\bar{t}_{kij}$).

In practice, it is too conservative to assume that all of the parameters with uncertainty can reach their extreme value simultaneously. Thus, we usually use an additional uncertainty set to cut the corner of the box set by taking the intersection of the two sets. For our problem, we adopt the so-called budget uncertainty set, which is given as follows:

$$\begin{aligned} \Phi_k \in U_k^{budget1} &= \{\Phi_k \in \mathbb{R}^{|L_k|} \mid \|\Phi_k\|_1 \leq \alpha_k\} = \left\{ \Phi_k \in \mathbb{R}^{|L_k|} \mid \sum_{(i,j) \in L_k} |\varphi_{kij}| \leq \alpha_k \right\} \\ \Psi_k \in U_k^{budget2} &= \{\Psi_k \in \mathbb{R}^{|L_k|} \mid \|\Psi_k\|_1 \leq \beta_k\} = \left\{ \Psi_k \in \mathbb{R}^{|L_k|} \mid \sum_{(i,j) \in L_k} |\psi_{kij}| \leq \beta_k \right\} \end{aligned}$$

where $\|\Phi_k\|_1 = \sum_{(i,j) \in L_k} |\varphi_{kij}|$ and $\|\Psi_k\|_1 = \sum_{(i,j) \in L_k} |\psi_{kij}|$ are the 1-norms of Φ_k and Ψ_k , respectively. α_k and β_k are the predefined upper bounds (e.g., the uncertainty budget) of the sum of the absolute values of φ_{kij} and ψ_{kij} , respectively. The uncertainty level of the budget uncertainty set can be represented by the ratio of the uncertainty budget and the corresponding total number of parameters with uncertainty (i.e., $\alpha_k/|L_k|$ and $\beta_k/|L_k|$). The intersection uncertainty set is given by the following:

$$\begin{aligned} \Phi_k \in U_k^1 &= U_k^{box1} \cap U_k^{budget1} = \{\Phi_k \in \mathbb{R}^{|L_k|} \mid \|\Phi_k\|_\infty \leq 1, \|\Phi_k\|_1 \leq \alpha_k\} \\ \Psi_k \in U_k^2 &= U_k^{box2} \cap U_k^{budget2} = \{\Psi_k \in \mathbb{R}^{|L_k|} \mid \|\Psi_k\|_\infty \leq 1, \|\Psi_k\|_1 \leq \beta_k\} \end{aligned}$$

whose uncertainty level is determined by the combination of the uncertainty level of the box uncertainty set and that of the budget uncertainty set.

3.3 Robust counterpart

In this part, we introduce the robust counterpart of the proposed deterministic model and demonstrate that the traditional robust counterpart is too conservative for our problem.

According to Ben-Tal et al. (2009), the so-called robust counterpart (RC) of the deterministic model S1 can be obtained by replacing constraints (36) and (37), which are influenced by the parameters with uncertainty, with the following constraints:

$$\begin{aligned} e_{kj} \leq e_{ki} - \left(\bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix} \right) - c_{kij}^{unit} e_k^{max} + p(\bar{t}_{kij} + \psi_{kij} \hat{t}_{kij}) x_{ij}, \\ \forall k \in K, \forall (i, j) \in L_k, \forall \varphi_{kij} \in U_{kij}^1, \forall \psi_{kij} \in U_{kij}^2 \end{aligned} \quad (41)$$

$$\begin{aligned} e_{kj} \geq e_{ki} - \left(\bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix} \right) - c_{kij}^{unit} e_k^{max} \\ \forall k \in K, \forall (i, j) \in L_k, \forall \varphi_{kij} \in U_{kij}^1 \end{aligned} \quad (42)$$

where $U_{kij}^1 = \{\varphi_{kij} \mid \Phi_k \in U_k^1\}$ and $U_{kij}^2 = \{\psi_{kij} \mid \Psi_k \in U_k^2\}$ denote the respective projections of U_k^1 and U_k^2 on the space of data of the constraint (36) corresponding to link $(i, j) \in L_k$. These can be easily obtained as follows:

$$\begin{aligned} \varphi_{kij} \in U_{kij}^1 &= \{\varphi_{kij} \mid -1 \leq \varphi_{kij} \leq 1, |\varphi_{kij}| \leq \alpha_k\} \\ \psi_{kij} \in U_{kij}^2 &= \{\psi_{kij} \mid -1 \leq \psi_{kij} \leq 1, |\psi_{kij}| \leq \beta_k\} \end{aligned}$$

Usually, $\alpha_k, \beta_k \in [1, |L_k|]$. Thus, U_{kij}^1 and U_{kij}^2 degrade to the following sets:

$$\begin{aligned}\varphi_{kij} &\in U_{kij}^1 = \{\varphi_{kij} \mid -1 \leq \varphi_{kij} \leq 1\} \\ \psi_{kij} &\in U_{kij}^2 = \{\psi_{kij} \mid -1 \leq \psi_{kij} \leq 1\}\end{aligned}$$

which are identical to the respective projections of box uncertainty sets U_k^{box1} and U_k^{box2} . We can observe that because the linkage between different constraints is broken by the projection process, the budget uncertainty sets become ineffective. Thus, the traditional robust counterpart will give us the most conservative solution, which corresponds to the condition that all of the parameters with uncertainty reach their worst-case value simultaneously. Note that the worst-case scenario for energy consumption parameters occurs when they all reach the largest value, while the worst-case scenario for possible charging time parameters occurs when they all reach their smallest value simultaneously.

In order to provide a less conservative robust formulation, we consider the more advanced concept of Adjustable Robust Counterpart (ARC).

3.4 Adjustable robust counterpart

In order to address the conservatism of RC in some application, Ben-Tal et al. (2004) developed a more advanced concept of ARC. In RC, there is an assumption that all decision variables represent “here and now” decisions, and they should be assigned specific numerical values as a result of solving the problem before the actual data “reveals itself” (Ben-Tal et al., 2009). As a relaxation of this assumption, the ARC allows some of the decision variables, which include auxiliary variables (e.g., slack or surplus variables) and variables representing “wait and see” decisions (i.e., decisions that can be made when part of the uncertain data become known) (Ben-Tal et al., 2004), to be adjustable based on different realizations of uncertain data through introducing functional relationships between decision variables and uncertain data. Thus, the optimal solution of an adjustable variable will be a determinate function of uncertain data rather than a single value. The value of an adjustable variable will not be determined until the actual value of uncertain data reveals itself.

In our deterministic model S1, the decision variables include x_{ij} , y_i , z_i , e_{ki} , and e_k^{max} . x_{ij} , y_i , and z_i indicate the locations of power transmitters. e_k^{max} represents the battery size of each electric bus line. e_{ki} denotes the battery level of an electric bus on line k at node i . From the perspective of system planning, the location of each DWPT facility and the battery size of every electric bus should be determined before building a DWPT electric bus system. Thus, decision variables x_{ij} , y_i , z_i , and e_k^{max} should represent “here and now” decisions and should not be adjustable variables. However, the variable e_{ki} , which denotes the battery level of an electric bus on line k after the bus traverses all links from the base station to node i , should represent a “wait and see” decision, because given the uncertainty of energy consumption and possible charging time, the battery level of an electric bus on line k at node i should be dependent on the actual energy consumption and charging time of every link passed rather than a predetermined value. Hence, variable e_{ki} should be an adjustable variable.

To obtain the ARC of our problem, the adjustable variable e_{ki} is to be replaced by a function of uncertain data. As discussed above, e_{ki} should be based on the uncertain data of all links passed (i.e., part of the uncertain data). Let L_k^i , where $i \in N_k$, denote the set of all of the links from the starting point of base station of line k , along the route of line k , to node i . Let Φ_k^i denote the vector of $\{\dots, \varphi_{kmn}, \dots\}$, and Ψ_k^i denote the vector of $\{\dots, \psi_{kmn}, \dots\}$, where $(m, n) \in L_k^i$. Note that every element φ_{kmn} in vector Φ_k^i is also an element in vector Φ_k , and every element ψ_{kmn} in vector Ψ_k^i is also an element in vector Ψ_k , namely, that Φ_k^i and Ψ_k^i are respective projections of Φ_k and Ψ_k from the space of $\mathbb{R}^{|L_k|}$ to the space of $\mathbb{R}^{|L_k^i|}$. Let $e_{ki}(\Phi_k^i, \Psi_k^i)$ denote the functional relationship between the adjustable variable e_{ki} and the uncertain data Φ_k^i and Ψ_k^i . The ARC of our problem then can be obtained by replacing constraints (35), (36), (37), (38) and (39) in S1 with the following constraints.

$$e_{ki}(\Phi_k^i, \Psi_k^i) = \epsilon_k^{lo} e_k^{max} \quad \forall k \in K, \forall i = O_k^s, \forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i} \quad (43)$$

$$e_{kj}(\Phi_k^j, \Psi_k^j) \leq e_{ki}(\Phi_k^i, \Psi_k^i) - (\bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix}) - c_{kij}^{unit} e_k^{max} + p(\bar{t}_{kij} + \psi_{kij} \hat{t}_{kij}) x_{ij} \quad \begin{array}{l} \forall k \in K, \forall (i, j) \in L_k \\ \forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i} \\ \forall \Phi_k^j \in U_k^{1j}, \forall \Psi_k^j \in U_k^{2j} \end{array} \quad (44)$$

$$e_{kj}(\Phi_k^j, \Psi_k^j) \geq e_{ki}(\Phi_k^i, \Psi_k^i) - (\bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix}) - c_{kij}^{unit} e_k^{max} \quad \begin{array}{l} \forall k \in K, \forall (i, j) \in L_k \\ \forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i} \\ \forall \Phi_k^j \in U_k^{1j}, \forall \Psi_k^j \in U_k^{2j} \end{array} \quad (45)$$

$$e_{ki}(\Phi_k^i, \Psi_k^i) \leq \epsilon_k^{up} e_k^{max} \quad \forall k \in K, \forall i \in N_k, \forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i} \quad (46)$$

$$e_{ki}(\Phi_k^i, \Psi_k^i) \geq \epsilon_k^{lo} e_k^{max} \quad \forall k \in K, \forall i \in N_k, \forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i} \quad (47)$$

where $U_k^{1i} = \{\Phi_k^i \in \mathbb{R}^{|L_k^i|} | \Phi_k \in U_k^1\}$ and $U_k^{2i} = \{\Psi_k^i \in \mathbb{R}^{|L_k^i|} | \Psi_k \in U_k^2\}$ are respective projections of U_k^1 and U_k^2 on the space of data of all links within L_k^i .

To obtain tractable ARC, [Ben-Tal et al. \(2004\)](#) suggested restricting the functional relationship between adjustable variables and uncertain data to be affine, namely, that $e_{ki}(\Phi_k^i, \Psi_k^i)$ is given as the following linear function:

$$e_{ki}(\Phi_k^i, \Psi_k^i) = \delta_k^i + \sum_{(m,n) \in L_k^i} \lambda_{kmn}^i \varphi_{kmn} + \sum_{(m,n) \in L_k^i} \mu_{kmn}^i \psi_{kmn} \quad (48)$$

where δ_k^i , λ_{kmn}^i , and μ_{kmn}^i are new decision variables that are nonadjustable.

Substituting all $e_{ki}(\Phi_k^i, \Psi_k^i)$ in ARC by Eq. (48) gives the so-called affinely adjustable robust counterpart (AARC). For completeness, we repeat some previously presented constraints here.

$$(S - AARC): \min_{(x_{ij}, y_i, z_i, \epsilon_k^{max}, \delta_k^i, \lambda_{kmn}^i, \mu_{kmn}^i)} a^{fix} \left\{ \sum_{i \in N} y_i - \sum_{i \in N^s} \sum_{(m,i) \in L} x_{mi} + \sum_{i \in N^s} z_i \right\} + a^{var} \sum_{(i,j) \in L} d_{ij} x_{ij} \\ + a^{bat} \sum_{k \in K} \zeta_k e_k^{max}$$

s. t.

$$y_i \leq \sum_{(i,j) \in L_i^+} x_{ij}, \quad \forall i \in N \quad (49)$$

$$y_i \leq 1 - x_{mi}, \quad \forall i \in N, \forall (m,i) \in L_i^- \quad (50)$$

$$y_i \geq x_{ij} - \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N, \forall (i,j) \in L_i^+ \quad (51)$$

$$z_i \leq \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N^s \quad (52)$$

$$z_i \geq x_{mi} \quad \forall i \in N^s, \forall (m,i) \in L_i^- \quad (53)$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in L \quad (54)$$

$$y_i \in \{0,1\} \quad \forall i \in N \quad (55)$$

$$z_i \in \{0,1\} \quad \forall i \in N^s \quad (56)$$

$$\delta_k^i = \epsilon_k^{up} e_k^{max} \quad \forall k \in K, \forall i = O_k^s \quad (57)$$

$$\delta_k^j + \sum_{(m,n) \in L_k^j} \lambda_{kmn}^j \varphi_{kmn} + \sum_{(m,n) \in L_k^j} \mu_{kmn}^j \psi_{kmn} \quad \begin{array}{l} \forall k \in K, \forall (i,j) \in L_k \\ \forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i} \\ \forall \Phi_k^j \in U_k^{1j}, \forall \Psi_k^j \in U_k^{2j} \end{array} \quad (58)$$

$$\begin{aligned}
&\leq \delta_k^i + \sum_{(m,n) \in L_k^i} \lambda_{kmn}^i \varphi_{kmn} + \sum_{(m,n) \in L_k^i} \mu_{kmn}^i \psi_{kmn} \\
&\quad - (\bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix}) - c_{kij}^{unit} e_k^{max} + p(\bar{t}_{kij} + \psi_{kij} \hat{t}_{kij}) x_{ij} \\
&\quad \delta_k^j + \sum_{(m,n) \in L_k^j} \lambda_{kmn}^j \varphi_{kmn} + \sum_{(m,n) \in L_k^j} \mu_{kmn}^j \psi_{kmn} \\
&\geq \delta_k^i + \sum_{(m,n) \in L_k^i} \lambda_{kmn}^i \varphi_{kmn} + \sum_{(m,n) \in L_k^i} \mu_{kmn}^i \psi_{kmn} \\
&\quad - (\bar{c}_{kij}^{fix} + \varphi_{kij} \hat{c}_{kij}^{fix}) - c_{kij}^{unit} e_k^{max}
\end{aligned} \tag{59}$$

$\forall k \in K, \forall (i, j) \in L_k$
 $\forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i}$
 $\forall \Phi_k^j \in U_k^{1j}, \forall \Psi_k^i \in U_k^{2j}$

$$\delta_k^i + \sum_{(m,n) \in L_k^i} \lambda_{kmn}^i \varphi_{kmn} + \sum_{(m,n) \in L_k^i} \mu_{kmn}^i \psi_{kmn} \leq \epsilon_k^{up} e_k^{max} \tag{60}$$

$\forall k \in K, \forall i \in N_k,$
 $\forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i}$

$$\delta_k^i + \sum_{(m,n) \in L_k^i} \lambda_{kmn}^i \varphi_{kmn} + \sum_{(m,n) \in L_k^i} \mu_{kmn}^i \psi_{kmn} \geq \epsilon_k^{lo} e_k^{max} \tag{61}$$

$\forall k \in K, \forall i \in N_k,$
 $\forall \Phi_k^i \in U_k^{1i}, \forall \Psi_k^i \in U_k^{2i}$

$$e_k^{max} > 0 \tag{62}$$

$\forall k \in K$

Note that for constraint (57), since $i = O_k^s$, L_k^i is a null set, $e_{ki}(\Phi_k^i, \Psi_k^i)$ equals δ_k^i . In the above formulation S-AARC, there are a finite number of variables and an infinite number of constraints. Thus, S-AARC is a semi-infinite programming problem, which is intractable.

3.5 Tractable and equivalent reformulation of S-AARC

All constraints (58), (59), (60) and (61) have a continuum of constraints, and it makes S-AARC intractable. Rearrange constraints (58), (59), (60) and (61) as follows:

$$\begin{aligned}
&\sum_{(m,n) \in L_k^i} (\lambda_{kmn}^j - \lambda_{kmn}^i) \varphi_{kmn} + (\lambda_{kij}^j + \hat{c}_{kij}^{fix}) \varphi_{kij} + \sum_{(m,n) \in L_k \setminus L_k^j} 0 \times \varphi_{kmn} \\
&+ \sum_{(m,n) \in L_k^i} (\mu_{kmn}^j - \mu_{kmn}^i) \psi_{kmn} + (\mu_{kij}^j - p \hat{t}_{kij} x_{ij}) \psi_{kij} + \sum_{(m,n) \in L_k \setminus L_k^j} 0 \times \psi_{kmn} \\
&\leq \delta_k^i - \delta_k^j - \bar{c}_{kij}^{fix} - c_{kij}^{unit} e_k^{max} + p \bar{t}_{kij} x_{ij}
\end{aligned} \tag{62}$$

$\forall k \in K, \forall (i, j) \in L_k$
 $\forall \Phi_k \in U_k^1, \forall \Psi_k \in U_k^2$

$$\begin{aligned}
&\sum_{(m,n) \in L_k^i} (\lambda_{kmn}^i - \lambda_{kmn}^j) \varphi_{kmn} + (-\lambda_{kij}^j - \hat{c}_{kij}^{fix}) \varphi_{kij} + \sum_{(m,n) \in L_k \setminus L_k^j} 0 \times \varphi_{kmn} \\
&+ \sum_{(m,n) \in L_k^i} (\mu_{kmn}^i - \mu_{kmn}^j) \psi_{kmn} + (-\mu_{kij}^j) \psi_{kij} + \sum_{(m,n) \in L_k \setminus L_k^j} 0 \times \psi_{kmn} \\
&\leq \delta_k^j - \delta_k^i + \bar{c}_{kij}^{fix} + c_{kij}^{unit} e_k^{max}
\end{aligned} \tag{63}$$

$\forall k \in K, \forall (i, j) \in L_k$
 $\forall \Phi_k \in U_k^1, \forall \Psi_k \in U_k^2$

$$\begin{aligned}
&\sum_{(m,n) \in L_k^i} \lambda_{kmn}^i \varphi_{kmn} + \sum_{(m,n) \in L_k \setminus L_k^i} 0 \times \varphi_{kmn} + \sum_{(m,n) \in L_k^i} \mu_{kmn}^i \psi_{kmn} + \sum_{(m,n) \in L_k \setminus L_k^i} 0 \times \psi_{kmn} \\
&\leq \epsilon_k^{up} e_k^{max} - \delta_k^i
\end{aligned} \tag{64}$$

$\forall k \in K, \forall i \in N_k$
 $\forall \Phi_k \in U_k^1, \forall \Psi_k \in U_k^2$

$$\begin{aligned}
& \sum_{(m,n) \in L_k^i} (-\lambda_{kmn}^i) \varphi_{kmn} + \sum_{(m,n) \in L_k \setminus L_k^i} 0 \times \varphi_{kmn} \\
& + \sum_{(m,n) \in L_k^i} (-\mu_{kmn}^i) \psi_{kmn} + \sum_{(m,n) \in L_k \setminus L_k^i} 0 \times \psi_{kmn} \\
& \leq -\epsilon_k^{lo} e_k^{max} + \delta_k^i
\end{aligned}$$

$$\begin{aligned}
& \forall k \in K, \forall i \in N_k \\
& \forall \Phi_k \in U_k^1, \forall \Psi_k \in U_k^2
\end{aligned} \quad (65)$$

where $L_k \setminus L_k^i = \{(m,n) | (m,n) \in L_k, (m,n) \notin L_k^i\}$. Note that we include uncertain parameters φ_{kmn} and ψ_{kmn} of all links $(m,n) \in L_k$ in each piece of constraint. For those parameters φ_{kmn} and ψ_{kmn} that should not appear, coefficients 0 are assigned to them.

Let V denote the vector including all the variables λ_{kmn}^i , μ_{kmn}^i , δ_k^i , and x_{ij} ($k \in K, i \in N_k, (m,n) \in L_k, (i,j) \in L_k$). Let $f(V)$, $g(V)$, and $h(V)$ denote affine functions of V . Each piece of constraint in (62), (63), (64) and (65) (i.e., for a certain electric bus line $k \in K$, and for a certain link $(i,j) \in L_k$ or a certain node $i \in N_k$) can be represented with the following general form:

$$\sum_{(m,n) \in L_k} \varphi_{kmn} f_{kmn}^i(V) + \sum_{(m,n) \in L_k} \psi_{kmn} g_{kmn}^i(V) \leq h_k^i(V) \quad \forall \Phi_k \in U_k^1, \forall \Psi_k \in U_k^2 \quad (66)$$

where $f_{kmn}^i(V)$ and $g_{kmn}^i(V)$ correspond to the coefficients of φ_{kmn} and ψ_{kmn} , respectively, and $h_k^i(V)$ represent the right hand side values. They will have different forms for constraints (62), (63), (64) and (65). Note that constraints (62) and (63) are link based constraints, we let the superscript i in constraint (66) denote the start node of link $(i,j) \in L_k$.

Here, we state and prove an equivalent reformation of constraint (66).

Proposition 1. *Constraint (66) is equivalent to the following system of constraints.*

$$\sum_{(m,n) \in L_k} \gamma_{kmn}^{i1} + \alpha_k \gamma_k^{i2} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i3} + \beta_k \gamma_k^{i4} \leq h_k^i(V) \quad (67)$$

$$\omega_{kmn}^{i1} + \omega_{kmn}^{i2} = f_{kmn}^i(V) \quad \forall (m,n) \in L_k \quad (68)$$

$$\omega_{kmn}^{i3} + \omega_{kmn}^{i4} = g_{kmn}^i(V), \quad \forall (m,n) \in L_k \quad (69)$$

$$-\gamma_{kmn}^{i1} \leq \omega_{kmn}^{i1} \leq \gamma_{kmn}^{i1} \quad \forall (m,n) \in L_k \quad (70)$$

$$-\gamma_k^{i2} \leq \omega_{kmn}^{i2} \leq \gamma_k^{i2} \quad \forall (m,n) \in L_k \quad (71)$$

$$-\gamma_{kmn}^{i3} \leq \omega_{kmn}^{i3} \leq \gamma_{kmn}^{i3} \quad \forall (m,n) \in L_k \quad (72)$$

$$-\gamma_k^{i4} \leq \omega_{kmn}^{i4} \leq \gamma_k^{i4} \quad \forall (m,n) \in L_k \quad (73)$$

where ω_{kmn}^{i1} , ω_{kmn}^{i2} , ω_{kmn}^{i3} and ω_{kmn}^{i4} are dual variables; γ_{kmn}^{i1} , γ_k^{i2} , γ_{kmn}^{i3} and γ_k^{i4} are auxiliary variables.

Proof. The equivalence can be proved using the duality theory. See Appendix A for the proof.

Each piece of constraints in (62), (63), (64), and (65) in problem S-AARC, after being reformulated as the general form (66), can be equivalently replaced by a system of constraints (67) to (73), which obviously have finite number of constraints. Thus, the original semi-infinite programming problem (S-AARC), which is intractable, can be equivalently reformulated as a tractable mathematical programming problem. The tractable reformulation of S-AARC, denoted as S-AARC-T, is provided in Appendix B. S-AARC-T is a mixed integer linear programming (MILP) problem, and it can be easily solved by commercial solvers such as CPLEX 12.1 (IBM ILOG, 2009).

4. Numerical study

To demonstrate the effectiveness of the proposed models, two numerical studies are presented. The first case study is based on the campus bus system of Utah State University (USU) in Logan, Utah, United States. The second case study is based on the bus system of downtown Salt Lake City (SLC), Utah, United States.

4.1 The bus systems

The campus bus system of Utah State University

Fig. 5(a) shows the routes of the campus bus system of USU. In total, there are four lines operating in the bus system. Assume that the university wants to transform this bus system to a DWPT electric bus system, in which case the location of DWPT facilities and the battery size of each electric bus need to be optimally determined. Four lines share the same base station; the red line and the green line operate clockwise; and the blue line and the purple line operate counter-clockwise. The network representation of the bus system can be then obtained, as shown in Fig. 5(b). The service loop and the number of buses of each line are given in Table 3. Four lines share the same base station, which is represented by node 1 and node 0. Electric buses start each service loop from node 1 and return to node 0 after finishing each service loop. Note that each link in Fig. 5 (b) will be further divided into short links in our model.

“place Fig. 5 about here”

“place Table 3 about here”

The Bus System of Salt Lake City

Fig. 6(a) shows the routes of the bus system we considered in downtown SLC. Totally, the bus system includes 8 bus lines (i.e., line 2, 2X, 3, 6, 11, 500, 519, 520). The simplified network representation of the bus system is shown in Fig. 6(b). Eight lines in the system share a base station at node 1. Table 4 shows the service loop and the number of buses for each line. The number of buses on each line is obtained based on the actual data of the SLC bus system.

“place Fig. 6 about here”

“place Table 4 about here”

4.2 Parameters of the deterministic model

The total length of all road segments in the USU campus bus system is 1.24 kilometers. The network is divided into 248 links. The SLC bus system covers 91.4 kilometers of road segments and is divided into 457 links. To evaluate the energy consumption on each link for each bus line, we need to determine the parameters in the energy consumption model. Table 5 shows a summary of the parameters we used in our model. For simplicity, we assume that all roads in the two networks have the same friction factor, that all bus lines use the same type of electric buses, and that the fixed part of total mass w_{kij}^{fix} is constant. The slope θ_{ij} is calculated based on the Digital Elevation Model (DEM) data from the Utah Automated Geographic Reference Center (AGRC). For a link beyond the influence of stations, stop signs and sidewalks, we assume that the acceleration rate of an electric bus on the link is zero, and the average speed is equal to the speed limit on the link. For a link within the influence of stations, stop signs and sidewalks, we assume that an electric bus on the link has a constant acceleration and deceleration rate with the value of 0.27ε , which is the comfortable deceleration rate defined by Highway Capacity Manual (HCM) (TRB, 2010), and that the average speed can be calculated through dividing link length by travel time. Moreover, we assume that each electric bus will always stop at its bus station for 50 seconds and will always decelerate to stop at stop signs and sidewalks. Note that the speed profile of each bus line is assumed to be predefined in our deterministic model. Based on all of these parameters, we can calculate the fixed part of energy consumption on each link for each electric bus line. The weight of battery pack per unit capacity is calculated based on the data from Bi et al. (2015). The parameters regarding DWPT facilities and batteries

are given in Table 6. Note that the service life of DWPT facilities is assumed to be 30 years, and the battery life is assumed to be two years. The cost of DWPT facilities and batteries is the amortized cost. Note that, when calculating the amortized cost, the discount rate should be considered. For simplicity, we assume that the discount rate and the battery price are constant over time. Let q denote the discount rate and let s denote the battery price per unit capacity. Then the amortized battery price a^{bat} is calculated as follows:

$$a^{bat} = \frac{1}{30} \sum_{\tau \in \{1,3,5,\dots,29\}} \frac{s}{(1+q)^{\tau-1}}$$

The battery price and the discount rate are assumed to be \$230/kWh and 0.01, respectively, and the amortized battery price is calculated to be \$100/kWh. The DWPT facilities are deployed before the operation of an electric bus system. For simplicity, we assume that the investment of the DWPT facilities is implemented in the first year of the service life. Thus, we can ignore the impact of the discount rate on the amortized cost of DWPT facilities. Moreover, we further assume that all electric buses use the same type of batteries.

“place Table 5 about here”

“place Table 6 about here”

4.3 Results of the deterministic models

Based on the network of the bus system of USU, we obtain a model with 1,361 variables (501 binary variables) and 1,812 constraints. GAMS (Rosenthal, 2012) and CPLEX solver (IBM ILOG, 2009) are used to solve our model. It only takes less than one second to solve the model with a 0.001% relative optimality gap and the optimal solution is shown in Table 7. A total of 16 DWPT facilities are allocated in the bus network. The total length of DWPT facilities is 2,750 m, which is only about 22.2% of the total length of road segments in the bus network. Fig. 7 shows the specific location of each DWPT facility in the bus network. We can observe that the DWPT facilities are primarily located around bus stations and turning points where buses will stop for a while. This result is reasonable because the energy supply from a DWPT facility is proportional to the travel time of an electric bus on the DWPT facility. It is more efficient to build DWPT facilities around bus stations and stop signs. Note that there are two DWPT facilities that are built around intersections and have two separate starting points, but in our model, each of them will be treated as one DWPT facility. In addition, we can also observe from Fig. 7 that there are four DWPT facilities shared by two bus lines and one DWPT facility shared by all four bus lines.

The total cost for the DWPT electric bus system is \$2,731,724. In our model, the total cost includes the cost of DWPT facilities and the total cost of batteries. The cost of building 16 power transmitters of 2,750 m long is \$870,000. The battery on each bus must be replaced with a new battery every two years. The total battery cost in 30 years is \$1,861,724.

“place Table 7 about here”

“place Fig. 7 about here”

To demonstrate the economic benefits of implementing the DWPT technique in an electric bus system, we compare the minimum total cost of building a DWPT electric bus system at USU with that of building a traditional stationary charging electric bus system. By solving the deterministic optimization model of a DWPT electric bus system with given parameters, we can obtain the optimal design of battery sizes for a stationary charging electric bus system. Table 8 shows the comparison of battery sizes between the DWPT electric bus system and the stationary charging electric bus system, and Table 9 shows the total cost comparison. Note that the cost of stationary charging facilities is not considered because both systems require stationary chargers at the base station. Table 8 indicates that all four lines in the DWPT electric bus system have a smaller battery size than in the stationary charging electric bus system. Table 9 shows that the total cost of a stationary charging electric bus system is \$3,432,497, whereas the total cost of the DWPT electric bus system is \$2,731,724. With the implementation of DWPT facilities, the DWPT electric bus system could reduce the total cost of the stationary charging electric bus system by 20.4%. Although the

DWPT electric bus system requires additional investments in DWPT infrastructure, its battery cost is much lower than the stationary charging electric bus system.

We further solve the deterministic model for the SLC bus system. The model has 1690 variables (937 binary variables) and 3707 constraints. It only takes 4.98 seconds to solve the model with a 0.001% relative optimality gap. The total cost for the DWPT electric bus system is \$9,248,331, including the \$3,780,000 cost for DWPT facilities and the \$5,468,331 cost for 30 years of batteries.

“place Table 8 about here”

“place Table 9 about here”

4.4 Uncertainty set of the robust model

As introduced in Section 3.2, the uncertainty set in our robust model is the intersection of the box uncertainty set and the budget uncertainty set. The uncertainty level is determined by the combination of the uncertainty level of the box uncertainty set and that of the budget uncertainty set. For simplicity, we assume that all four bus lines have the same uncertainty level. The ratios $\hat{c}_{kij}^{fix}/\bar{c}_{kij}^{fix}$ and $\hat{t}_{kij}/\bar{t}_{kij}$, which determine the respective uncertainty level of the box uncertainty set for energy consumption and travel time, are assigned the same value. In addition, the ratios $\alpha_k/|L_k|$ and $\beta_k/|L_k|$, which determine the respective uncertainty level of the budget uncertainty set for energy consumption and travel time, are also set to be the same. Let χ^{box} and χ^{budget} denote the uncertainty level parameters of the box uncertainty set and the budget uncertainty set, respectively. χ^{box} and χ^{budget} are given by

$$\chi^{box} = \frac{\hat{c}_{kij}^{fix}}{\bar{c}_{kij}^{fix}} = \frac{\hat{t}_{kij}}{\bar{t}_{kij}} \quad \forall k \in K, \forall (i, j) \in L$$

$$\chi^{budget} = \frac{\alpha_k}{|L_k|} = \frac{\beta_k}{|L_k|} \quad \forall k \in K$$

For the USU campus bus system, to investigate the influence of the uncertainty level on the total cost and the optimal solution, we consider 121 groups of uncertainty level with values of χ^{box} and χ^{budget} separately ranging between 0 and 1 with a step size of 0.1. For the SLC bus system, we simply use one group of uncertainty level with both parameters χ^{box} and χ^{budget} being 0.1 to demonstrate the tractability of the robust model.

4.5 Results of the robust model

The robust model for the USU campus bus system has 613,265 variables and 934,890 constraints. Since it is still an MILP problem, we can use GAMS (Rosenthal, 2012) and CPLEX solver (IBM ILOG, 2009) to solve it. With a 0.5% relative optimality gap, the computation time is around 2 hours, depending on the uncertainty level parameters. For instance, when the uncertainty level parameters χ^{box} and χ^{budget} are both 0.1, the computation time for the corresponding robust model is 6241 seconds (1h 44min 1s). Table 10 shows the comparison between the results of one robust model with an uncertainty level of $\chi^{box} = 0.1$ and $\chi^{budget} = 1.0$ and the results of the deterministic model. To consider the uncertainty of energy consumption and possible charging time at the level of $\chi^{box} = 0.1$ and $\chi^{budget} = 1.0$, the total cost of the DWPT electric bus system will increase from \$2,731,724 to \$3,242,080. In the results of the robust model, all four lines require larger batteries than those required in the deterministic model. Moreover, the layout of DWPT facilities in the robust model is also different from that of the deterministic model.

“place Table 10 about here”

Although the robust optimal solution requires greater investments, the corresponding DWPT electric bus system can operate uninterrupted when energy consumption and possible charging times have deviations within the uncertainty set. Consider the worst-case scenario, in which all of the parameters pertaining to energy consumption and possible charging times have a 10 percent deviation rate from the expected value. With the solutions of the deterministic model and the solutions of the robust model, we can

obtain the corresponding battery level profiles of each bus line within one service loop. Fig. 8 shows the comparison of the battery level profile of the red (#1) bus line between the deterministic model solution and the robust model solution under the worst-case scenario. It is obvious that, in the worst-case scenario, the red (#1) line electric bus, under the robust model solution, can operate normally within the given range of the battery level. In contrast, under the deterministic model solution, the electric bus will use its battery beyond its given range. Thus, when the worst-case scenario occurs, three issues will arise for the DWPT electric bus system under the deterministic model solution. First, the battery life will be reduced due to usage beyond its given range. Second, the electric buses will need more charging time at the base station. And third, in an extreme case where the deviation of energy consumption and charging time from expected values is substantially large, the electric buses may run out of battery power before they return to the base station.

“place Fig. 8 about here”

With our robust model, we can obtain the optimal design for a DWPT electric bus system that is robust against the uncertainty of energy consumption and travel time. However, additional investments will be required when we seek the optimal robust design. For different uncertainty levels of the uncertainty set, the required cost will also be different. The different total costs of a DWPT electric bus system for 121 groups of different uncertainty levels, which correspond to the values of χ^{box} and χ^{budget} separately ranging between 0 and 1 with a step size of 0.1, are shown in the upper three dimensional plots in Fig. 9. The lower two plots in Fig. 9 show the same results with two dimensional plots. In the lower-left plot, the x -axes represents the uncertainty level of the budget uncertainty set, and the y -axes represents the total cost, with different uncertainty levels of the box uncertainty set given in different curves. In the lower-right plot, the x -axes represents the uncertainty level of the box uncertainty set, and the y -axes represents the total cost, with different uncertainty levels of the budget uncertainty set given in different curves. Based on the three plots in Fig. 9, we can gain some important insights into the robust optimal design of a DWPT electric bus system. First, the total cost of a DWPT electric bus system will increase along with the level of robustness, which is represented by the uncertainty level of the box uncertainty set and that of the budget uncertainty set. Second, when the uncertainty level of the box uncertainty set is given, as the increase of the uncertainty level of the budget uncertainty set, the total cost will increase at a decreasing rate. Third, when the uncertainty level of the budget uncertainty set is given, with the increase of the uncertainty level of the box uncertainty set, the total cost will increase and the increment is almost linear.

“place Fig. 9 about here”

In our robust model, the box uncertainty set determines the maximum deviation of each individual parameter with uncertainty. Thus, the influence of the uncertainty level of the box uncertainty set on the total cost is almost uniform. The budget uncertainty set in the robust model determines the maximum proportion of all parameters with uncertainty that can reach the worst-case value. Due to the lack of uniformity of the parameters with uncertainty, the increment rate of the total cost will decrease with the uncertainty level of the budget uncertainty set.

We further solve the robust model for the SLC bus system. The model has 1,267,446 variables (937 binary variables) and 2,101,663 constraints. With a 0.5% relative optimality gap, the computation time for the robust model is 16h 1min 55s. The total cost for the DWPT electric bus system is \$9,645,869, including the \$3,680,000 cost for DWPT facilities and the \$5,965,869 cost for 30 years of batteries.

5. Concluding remarks

In this paper, we address the robust planning problem of dynamic wireless charging infrastructure for battery electric buses. A MIP model is first formulated to optimize the battery size of each electric bus and the allocation of DWPT facilities of a DWPT electric bus system. The model is applicable to a general DWPT electric bus system with several overlapping bus lines. Given the uncertainty in terms of the energy consumption and travel time of electric buses, robust planning solutions are needed. With the robust optimization technique, we formulate the robust counterpart of the deterministic model and take into account the uncertainty of the energy consumption and travel time parameters. The intersection of the box

uncertainty set and the budget uncertainty set is assumed for modeling uncertain energy consumption and travel time. The concept of ARC is adopted in our model to obtain a less conservative robust model, and the AARC approach is adopted to derive a tractable reformulation of the robust model. Both the deterministic model and the robust model are tested with numerical examples. The results demonstrate that our deterministic model can effectively solve the planning problem of a DWPT electric bus system with several overlapping lines, and that the optimal design reduces the battery size as well as the total cost of the electric system dramatically. The comparison between the solutions of the deterministic model and those of the robust model under the worst-case scenario demonstrate that the RO approach provides solutions that are robust against parameter uncertainties. With different uncertainty levels, we investigate the relationship between the total cost and the level of robustness of a DWPT electric bus system. The results may help decision makers determine the best trade-off between investments and the level of robustness of a DWPT electric bus system.

The DWPT electric bus system, which is clean and sustainable, could be widely adopted in the near future. The proposed modeling framework in this study provides practitioners with an effective tool to determine the optimal allocation of DWPT facilities as well as the battery size of each bus line for a DWPT electric bus system.

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Appendix A: Proof of Proposition 1

Constraints (66) can be equivalently given by

$$\max_{\Phi_k \in U_k^1, \Psi_k \in U_k^2} \left\{ \sum_{(m,n) \in L_k} \varphi_{kmn} f_{kmn}^i(V) + \sum_{(m,n) \in L_k} \psi_{kmn} g_{kmn}^i(V) \right\} \leq h_k^i(V) \quad (\text{A.1})$$

The uncertainty sets U_k^1 and U_k^2 can be rewritten as the following equivalent conic representation

$$\begin{aligned} \Phi_k \in U_k^1 &= \{ \Phi_k \in \mathbb{R}^{|L_k|} | T_k^1 \Phi_k + t_k^1 \in W_k^1, T_k^2 \Phi_k + t_k^2 \in W_k^2 \} \\ \Psi_k \in U_k^2 &= \{ \Psi_k \in \mathbb{R}^{|L_k|} | T_k^3 \Psi_k + t_k^3 \in W_k^3, T_k^4 \Psi_k + t_k^4 \in W_k^4 \} \end{aligned}$$

where $T_k^1 \Phi_k \equiv [\Phi_k; 0]$, $t_k^1 = [0_{|L_k| \times 1}; 1]$, $T_k^2 \Phi_k \equiv [\Phi_k; 0]$, $t_k^2 = [0_{|L_k| \times 1}; \Gamma_k]$, $T_k^3 \Psi_k \equiv [\Psi_k; 0]$, $t_k^3 = [0_{|L_k| \times 1}; 1]$ and $W_k^1 = W_k^3 = \{[A; b] \in \mathbb{R}^{|L_k|} \times \mathbb{R} | b \geq \|A\|_\infty\}$; $T_k^2 \Phi_k \equiv [\Phi_k; 0]$, $t_k^2 = [0_{|L_k| \times 1}; \Gamma_k]$, $T_k^4 \Psi_k \equiv [\Psi_k; 0]$, $t_k^4 = [0_{|L_k| \times 1}; \Lambda_k]$ and $W_k^2 = W_k^4 = \{[A; b] \in \mathbb{R}^{|L_k|} \times \mathbb{R} | b \geq \|A\|_1\}$. W_k^1, W_k^2, W_k^3 and W_k^4 are all norm cones.

Thus, the left part of inequality (A.1) can be treated as a conic optimization problem (P).

$$(P) \quad \max_{\varphi_{kmn}, \psi_{kmn}} \sum_{(m,n) \in L_k} \varphi_{kmn} f_{kmn}^i(V) + \sum_{(m,n) \in L_k} \psi_{kmn} g_{kmn}^i(V)$$

s. t.

$$\begin{aligned} \Phi_k \in U_k^1 &= \{ \Phi_k \in \mathbb{R}^{|L_k|} | T_k^1 \Phi_k + t_k^1 \in W_k^1, T_k^2 \Phi_k + t_k^2 \in W_k^2 \} \\ \Psi_k \in U_k^2 &= \{ \Psi_k \in \mathbb{R}^{|L_k|} | T_k^3 \Psi_k + t_k^3 \in W_k^3, T_k^4 \Psi_k + t_k^4 \in W_k^4 \} \end{aligned}$$

According to the property of strong duality (Glineur, 2001, Chapter 4, Ben-Tal et al., 2009, Appendix A2), the equivalent dual problem (D) of (P) is given as follows:

$$(D) \quad \min_{\tau_k^{i1}, \tau_k^{i2}, \tau_k^{i3}, \tau_k^{i4}, \omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}, \omega_{kmn}^{i4}} \tau_k^{i1} + \alpha_k \tau_k^{i2} + \tau_k^{i3} + \beta_k \tau_k^{i4}$$

s. t.

$$(\Omega_k^{i1} + \Omega_k^{i2})_{mn} = \omega_{kmn}^{i1} + \omega_{kmn}^{i2} = f_{kmn}^i(V) \quad \forall (m, n) \in L_k \quad (\text{A.2})$$

$$(\Omega_k^{i3} + \Omega_k^{i4})_{mn} = \omega_{kmn}^{i3} + \omega_{kmn}^{i4} = g_{kmn}^i(V) \quad \forall (m, n) \in L_k \quad (\text{A.3})$$

$$[\Omega_k^{i1}; \tau_k^{i1}] \in W_k^{1*} \quad (\text{A.4})$$

$$[\Omega_k^{i2}; \tau_k^{i2}] \in W_k^{2*} \quad (\text{A.5})$$

$$[\Omega_k^{i3}; \tau_k^{i3}] \in W_k^{3*} \quad (\text{A.6})$$

$$[\Omega_k^{i4}; \tau_k^{i4}] \in W_k^{4*} \quad (\text{A.7})$$

Where $\tau_k^{i1}, \tau_k^{i2}, \tau_k^{i3}, \tau_k^{i4}, \omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}$, and ω_{kmn}^{i4} are dual variables; $\Omega_k^{i1} = \{\dots, \omega_{kmn}^{i1}, \dots\}$, $\Omega_k^{i2} = \{\dots, \omega_{kmn}^{i2}, \dots\}$, $\Omega_k^{i3} = \{\dots, \omega_{kmn}^{i3}, \dots\}$, $\Omega_k^{i4} = \{\dots, \omega_{kmn}^{i4}, \dots\}$; $W_k^{1*}, W_k^{2*}, W_k^{3*}, W_k^{4*}$ are the dual cones of $W_k^1, W_k^2, W_k^3, W_k^4$, respectively.

According to the conic duality theory (Glineur, 2001, Chapter 4 Theorem 4.3), $W_k^{1*}, W_k^{2*}, W_k^{3*}, W_k^{4*}$ are given as follows

$$W_k^{1*} = W_k^{3*} = W_k^2 = W_k^4 = \{[A; b] \in \mathbb{R}^{|L_k|} \times \mathbb{R} | b \geq \|A\|_1\}$$

$$W_k^{2*} = W_k^{4*} = W_k^1 = W_k^3 = \{[A; b] \in \mathbb{R}^{|L_k|} \times \mathbb{R} | b \geq \|A\|_\infty\}$$

Thus constraints (A.4), (A.5), (A.6), and (A.7) are given as follows:

$$\|\Omega_k^{i1}\|_1 \leq \tau_k^{i1} \quad (\text{A.8})$$

$$\|\Omega_k^{i2}\|_\infty \leq \tau_k^{i2} \quad (\text{A.9})$$

$$\|\Omega_k^{i3}\|_1 \leq \tau_k^{i3} \quad (\text{A.10})$$

$$\|\Omega_k^{i4}\|_\infty \leq \tau_k^{i4} \quad (\text{A.11})$$

Through eliminating variables $\tau_k^{i1}, \tau_k^{i2}, \tau_k^{i3}, \tau_k^{i4}$, the dual problem (D) can be reduced to the following problem (D1).

$$(D1) \quad \min_{\omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}, \omega_{kmn}^{i4}} \sum_{(m,n) \in L_k} |\omega_{kmn}^{i1}| + \alpha_k \max_{(m,n) \in L_k} |\omega_{kmn}^{i2}| + \sum_{(m,n) \in L_k} |\omega_{kmn}^{i3}|$$

$$+ \beta_k \max_{(m,n) \in L_k} |\omega_{kmn}^{i4}|$$

s. t.

$$\omega_{kmn}^{i1} + \omega_{kmn}^{i2} = f_{kmn}^i(V) \quad \forall (m, n) \in L_k \quad (\text{A.12})$$

$$\omega_{kmn}^{i3} + \omega_{kmn}^{i4} = g_{kmn}^i(V) \quad \forall (m, n) \in L_k \quad (\text{A.13})$$

Essentially, D1 is a linear programming problem. To eliminate the absolute value sign in its objective function, we introduce an auxiliary variable γ_{kmn}^{i1} for each ω_{kmn}^{i1} , an auxiliary variable γ_{kmn}^{i3} for each ω_{kmn}^{i3} , an auxiliary variable γ_k^{i2} for vector Ω_k^{i2} and an auxiliary variable γ_k^{i4} for vector Ω_k^{i4} , problem (D1) then can be equivalently represented by the following mathematical program (D2).

$$(D2) \quad \min_{\omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}, \omega_{kmn}^{i4}, \gamma_{kmn}^{i1}, \gamma_k^{i2}, \gamma_{kmn}^{i3}, \gamma_k^{i4}} \sum_{(m,n) \in L_k} \gamma_{kmn}^{i1} + \alpha_k \gamma_k^{i2} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i3} + \beta_k \gamma_k^{i4}$$

s. t.

$$\omega_{kmn}^{i1} + \omega_{kmn}^{i2} = f_{kmn}^i(V) \quad \forall (m, n) \in L_k \quad (\text{A.14})$$

$$\omega_{kmn}^{i3} + \omega_{kmn}^{i4} = g_{kmn}^i(V), \quad \forall (m, n) \in L_k \quad (\text{A.15})$$

$$-\gamma_{kmn}^{i1} \leq \omega_{kmn}^{i1} \leq \gamma_{kmn}^{i1} \quad \forall (m, n) \in L_k \quad (\text{A.16})$$

$$-\gamma_k^{i2} \leq \omega_{kmn}^{i2} \leq \gamma_k^{i2} \quad \forall (m, n) \in L_k \quad (\text{A.17})$$

$$-\gamma_{kmn}^{i3} \leq \omega_{kmn}^{i3} \leq \gamma_{kmn}^{i3} \quad \forall (m, n) \in L_k \quad (\text{A.18})$$

$$-\gamma_k^{i4} \leq \omega_{kmn}^{i4} \leq \gamma_k^{i4} \quad \forall (m, n) \in L_k \quad (\text{A.19})$$

Since problem (D2) is equivalent to (P), they will have the same optimal values. Therefore, constraint (A.1), which requires that the optimal value of (P) is $\leq h_k^i(V)$, is equivalent to that the optimal value of (D2) is achieved and is $\leq h_k^i(V)$, i.e., the following constraint:

$$\min_{\omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}, \omega_{kmn}^{i4}, \gamma_{kmn}^{i1}, \gamma_k^{i2}, \gamma_{kmn}^{i3}, \gamma_k^{i4}} \sum_{(m,n) \in L_k} \gamma_{kmn}^{i1} + \alpha_k \gamma_k^{i2} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i3} + \beta_k \gamma_k^{i4} \leq h_k^i(V) \quad (\text{A.20})$$

where the feasible region of $(\omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}, \omega_{kmn}^{i4}, \gamma_{kmn}^{i1}, \gamma_k^{i2}, \gamma_{kmn}^{i3}, \gamma_k^{i4})$ is given by constraints (A.14)-(A.19). Because (D2) is a minimization problem, constraint (A.20), which requires the optimal value of (D2) is achieved and is $\leq h_k^i(V)$, is equivalent to that (D2) has a feasible solution $(\omega_{kmn}^{i1}, \omega_{kmn}^{i2}, \omega_{kmn}^{i3}, \omega_{kmn}^{i4}, \gamma_{kmn}^{i1}, \gamma_k^{i2}, \gamma_{kmn}^{i3}, \gamma_k^{i4})$ with $\sum_{(m,n) \in L_k} \gamma_{kmn}^{i1} + \alpha_k \gamma_k^{i2} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i3} + \beta_k \gamma_k^{i4} \leq h_k^i(V)$. Therefore, based on the above discussions, constraint (66) can be equivalently replaced by the following system of constraints:

$$\sum_{(m,n) \in L_k} \gamma_{kmn}^{i1} + \alpha_k \gamma_k^{i2} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i3} + \beta_k \gamma_k^{i4} \leq h_k^i(V) \quad (\text{A.21})$$

$$\omega_{kmn}^{i1} + \omega_{kmn}^{i2} = f_{kmn}^i(V) \quad \forall (m,n) \in L_k \quad (\text{A.22})$$

$$\omega_{kmn}^{i3} + \omega_{kmn}^{i4} = g_{kmn}^i(V), \quad \forall (m,n) \in L_k \quad (\text{A.23})$$

$$-\gamma_{kmn}^{i1} \leq \omega_{kmn}^{i1} \leq \gamma_{kmn}^{i1} \quad \forall (m,n) \in L_k \quad (\text{A.24})$$

$$-\gamma_k^{i2} \leq \omega_{kmn}^{i2} \leq \gamma_k^{i2} \quad \forall (m,n) \in L_k \quad (\text{A.25})$$

$$-\gamma_{kmn}^{i3} \leq \omega_{kmn}^{i3} \leq \gamma_{kmn}^{i3} \quad \forall (m,n) \in L_k \quad (\text{A.26})$$

$$-\gamma_k^{i4} \leq \omega_{kmn}^{i4} \leq \gamma_k^{i4} \quad \forall (m,n) \in L_k \quad (\text{A.27})$$

where ω_{kmn}^{i1} , ω_{kmn}^{i2} , ω_{kmn}^{i3} and ω_{kmn}^{i4} are dual variables; γ_{kmn}^{i1} , γ_k^{i2} , γ_{kmn}^{i3} , and γ_k^{i4} are auxiliary variables.

Appendix B: Tractable reformulation of S-AARC

Each piece of constraints in (62), (63), (64), and (65) in (S-AARC), after being reformulated as the general form (66), can be equivalently replaced by a system of constraints (67) to (73), which obviously have finite number of constraints. Thus, the original semi-infinite programming problem (S-AARC), which is intractable, can be equivalently reformulated as the following tractable programming problem (S-AARC-T):

$$\begin{aligned} (\text{S-AARC-T}): \min_{x,y,z,e^{\max},\delta,\lambda,\mu,\omega,\gamma} \quad & a^{fix} \left\{ \sum_{i \in N} y_i - \sum_{i \in N^s} \sum_{(m,i) \in L} x_{mi} + \sum_{i \in N^s} z_i \right\} + a^{var} \sum_{(i,j) \in L} d_{ij} x_{ij} \\ & + a^{bat} \sum_{k \in K} \zeta_k e_k^{\max} \end{aligned}$$

s. t.

$$y_i \leq \sum_{(i,j) \in L_i^+} x_{ij}, \quad \forall i \in N \quad (\text{B.1})$$

$$y_i \leq 1 - x_{mi}, \quad \forall i \in N, \forall (m,i) \in L_i^- \quad (\text{B.2})$$

$$y_i \geq x_{ij} - \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N, \forall (i,j) \in L_i^+ \quad (\text{B.3})$$

$$z_i \leq \sum_{(m,i) \in L_i^-} x_{mi} \quad \forall i \in N^s \quad (\text{B.4})$$

$$z_i \geq x_{mi} \quad \forall i \in N^s, \forall (m,i) \in L_i^- \quad (\text{B.5})$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in L \quad (\text{B.6})$$

$$y_i \in \{0,1\} \quad \forall i \in N \quad (\text{B.7})$$

$$z_i \in \{0,1\} \quad \forall i \in N^s \quad (\text{B.8})$$

$$\delta_k^i = \epsilon_k^{up} e_k^{\max} \quad \forall k \in K, \forall i = O_k^s \quad (\text{B.9})$$

$$\begin{aligned} \sum_{(m,n) \in L_k} \gamma_{kmn}^{i1} + \alpha_k \gamma_k^{i2} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i3} + \beta_k \gamma_k^{i4} & \leq \delta_k^i - \delta_k^j - \bar{c}_{kij}^{fix} - c_{kij}^{unit} e_k^{\max} + p \bar{t}_{kij} x_{ij} \quad \forall k \in K, \forall (i,j) \in L_k \quad (\text{B.10}) \\ \omega_{kmn}^{i1} + \omega_{kmn}^{i2} = \lambda_{kmn}^j - \lambda_{kmn}^i & \quad \forall k \in K, \forall (i,j) \in L_k, \forall (m,n) \in L_k^i \quad (\text{B.11}) \end{aligned}$$

$$\omega_{kij}^{i1} + \omega_{kij}^{i2} = \lambda_{kij}^j + \hat{c}_{kij}^{fix} \quad \forall k \in K, \forall (i, j) \in L_k \quad (B.12)$$

$$\omega_{kmn}^{i1} + \omega_{kmn}^{i2} = 0 \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k \setminus L_k^j \quad (B.13)$$

$$\omega_{kmn}^{i3} + \omega_{kmn}^{i4} = \mu_{kmn}^j - \mu_{kmn}^i \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k^i \quad (B.14)$$

$$\omega_{kij}^{i3} + \omega_{kij}^{i4} = \mu_{kij}^j - p\hat{t}_{kij}x_{ij} \quad \forall k \in K, \forall (i, j) \in L_k \quad (B.15)$$

$$\omega_{kmn}^{i3} + \omega_{kmn}^{i4} = 0 \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k \setminus L_k^j \quad (B.16)$$

$$\omega_{kmn}^{i1} \leq \gamma_{kmn}^{i1} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.17)$$

$$-\omega_{kmn}^{i1} \leq \gamma_{kmn}^{i1} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.18)$$

$$\omega_{kmn}^{i2} \leq \gamma_k^{i2} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.19)$$

$$-\omega_{kmn}^{i2} \leq \gamma_k^{i2} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.20)$$

$$\omega_{kmn}^{i3} \leq \gamma_{kmn}^{i3} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.21)$$

$$-\omega_{kmn}^{i3} \leq \gamma_{kmn}^{i3} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.22)$$

$$\omega_{kmn}^{i4} \leq \gamma_k^{i4} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.23)$$

$$-\omega_{kmn}^{i4} \leq \gamma_k^{i4} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.24)$$

$$\sum_{(m,n) \in L_k} \gamma_{kmn}^{i5} + \alpha_k \gamma_k^{i6} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i7} + \beta_k \gamma_k^{i8} \quad \forall k \in K, \forall (i, j) \in L_k \quad (B.25)$$

$$\leq \delta_k^j - \delta_k^i + \bar{c}_{kij}^{fix} + c_{kij}^{unit} e_k^{max}$$

$$\omega_{kmn}^{i5} + \omega_{kmn}^{i6} = \lambda_{kmn}^j - \lambda_{kmn}^i \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k^i \quad (B.26)$$

$$\omega_{kij}^{i5} + \omega_{kij}^{i6} = -\lambda_{kij}^j - \hat{c}_{kij}^{fix} \quad \forall k \in K, \forall (i, j) \in L_k \quad (B.27)$$

$$\omega_{kmn}^{i5} + \omega_{kmn}^{i6} = 0 \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k \setminus L_k^j \quad (B.28)$$

$$\omega_{kmn}^{i7} + \omega_{kmn}^{i8} = \mu_{kmn}^i - \mu_{kmn}^j \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k^i \quad (B.29)$$

$$\omega_{kij}^{i7} + \omega_{kij}^{i8} = -\mu_{kij}^j \quad \forall k \in K, \forall (i, j) \in L_k \quad (B.30)$$

$$\omega_{kmn}^{i7} + \omega_{kmn}^{i8} = 0 \quad \forall k \in K, \forall (i, j) \in L_k, \forall (m, n) \in L_k \setminus L_k^j \quad (B.31)$$

$$\omega_{kmn}^{i5} \leq \gamma_{kmn}^{i5} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.32)$$

$$-\omega_{kmn}^{i5} \leq \gamma_{kmn}^{i5} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.33)$$

$$\omega_{kmn}^{i6} \leq \gamma_k^{i6} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.34)$$

$$-\omega_{kmn}^{i6} \leq \gamma_k^{i6} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.35)$$

$$\omega_{kmn}^{i7} \leq \gamma_{kmn}^{i7} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.36)$$

$$-\omega_{kmn}^{i7} \leq \gamma_{kmn}^{i7} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.37)$$

$$\omega_{kmn}^{i8} \leq \gamma_k^{i8} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.38)$$

$$-\omega_{kmn}^{i8} \leq \gamma_k^{i8} \quad \forall k \in K, \forall (m, n) \in L_k, \forall i \in N_k \setminus O_k^s, \quad (B.39)$$

$$\sum_{(m,n) \in L_k} \gamma_{kmn}^{i9} + \alpha_k \gamma_k^{i10} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i11} + \beta_k u_k^{i12} \quad \forall k \in K, \forall i \in N_k, \quad (B.40)$$

$$\leq \epsilon_k^{up} e_k^{max} - \delta_k^i$$

$$\omega_{kmn}^{i9} + \omega_{kmn}^{i10} = \lambda_{kmn}^i \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k^i \quad (B.41)$$

$$\omega_{kmn}^{i9} + \omega_{kmn}^{i10} = 0 \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \setminus L_k^i \quad (B.42)$$

$$\omega_{kmn}^{i11} + \omega_{kmn}^{i12} = \mu_{kmn}^i \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k^i \quad (B.43)$$

$$\omega_{kmn}^{i11} + \omega_{kmn}^{i12} = 0 \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \setminus L_k^i, \quad (B.44)$$

$$\omega_{kmn}^{i9} \leq \gamma_{kmn}^{i9} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (B.45)$$

$$-\omega_{kmn}^{i9} \leq \gamma_{kmn}^{i9} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (B.46)$$

$$\omega_{kmn}^{i10} \leq \gamma_k^{i10} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (B.47)$$

$$-\omega_{kmn}^{i10} \leq \gamma_k^{i10} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (B.48)$$

$$\omega_{kmn}^{i11} \leq \gamma_{kmn}^{i11} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (B.49)$$

$$-\omega_{kmn}^{i11} \leq \gamma_{kmn}^{i11} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.50})$$

$$\omega_{kmn}^{i12} \leq \gamma_k^{i12} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.51})$$

$$-\omega_{kmn}^{i12} \leq \gamma_k^{i12} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.52})$$

$$\sum_{(m,n) \in L_k} \gamma_{kmn}^{i13} + \alpha_k \gamma_k^{i14} + \sum_{(m,n) \in L_k} \gamma_{kmn}^{i15} + \beta_k \gamma_k^{i16} \quad \forall k \in K, \forall i \in N_k, \quad (\text{B.53})$$

$$\leq -\epsilon_k^{lo} e_k^{max} + \delta_k^i$$

$$\omega_{kmn}^{i13} + \omega_{kmn}^{i14} = -\lambda_{kmn}^i \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k^i \quad (\text{B.54})$$

$$\omega_{kmn}^{i13} + \omega_{kmn}^{i14} = 0 \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \setminus L_k^i \quad (\text{B.55})$$

$$\omega_{kmn}^{i15} + \omega_{kmn}^{i16} = -\mu_{kmn}^i \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k^i \quad (\text{B.56})$$

$$\omega_{kmn}^{i15} + \omega_{kmn}^{i16} = 0 \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \setminus L_k^i, \quad (\text{B.57})$$

$$\omega_{kmn}^{i13} \leq \gamma_{kmn}^{i13} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.58})$$

$$-\omega_{kmn}^{i13} \leq \gamma_{kmn}^{i13} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.59})$$

$$\omega_{kmn}^{i14} \leq \gamma_k^{i14} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.60})$$

$$-\omega_{kmn}^{i14} \leq \gamma_k^{i14} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.61})$$

$$\omega_{kmn}^{i15} \leq \gamma_{kmn}^{i15} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.62})$$

$$-\omega_{kmn}^{i15} \leq \gamma_{kmn}^{i15} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.63})$$

$$\omega_{kmn}^{i16} \leq \gamma_k^{i16} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.64})$$

$$-\omega_{kmn}^{i16} \leq \gamma_k^{i16} \quad \forall k \in K, \forall i \in N_k, \forall (m, n) \in L_k \quad (\text{B.65})$$

$$e_k^{max} > 0 \quad \forall k = 1, 2, \dots, M \quad (\text{B.66})$$

where the dual variables, ω_{kmn}^{i1} , ω_{kmn}^{i2} , ω_{kmn}^{i3} , and ω_{kmn}^{i4} , auxiliary variables, γ_{kmn}^{i1} , γ_k^{i2} , γ_{kmn}^{i3} , and γ_k^{i4} , and constraints (B.10)-(B.24) correspond to constraints (58) in the original (S-AARC); the dual variables, ω_{kmn}^{i5} , ω_{kmn}^{i6} , ω_{kmn}^{i7} , and ω_{kmn}^{i8} , auxiliary variables, γ_{kmn}^{i5} , γ_k^{i6} , γ_{kmn}^{i7} , and γ_k^{i8} and constraints (B.25)-(B.39) correspond constraints (59); the dual variables, ω_{kmn}^{i9} , ω_{kmn}^{i10} , ω_{kmn}^{i11} , and ω_{kmn}^{i12} , auxiliary variables, γ_{kmn}^{i9} , γ_k^{i10} , γ_{kmn}^{i11} , and γ_k^{i12} , and constraints (B.40)-(B.52) correspond the original constraints (60); and the dual variables, ω_{kmn}^{i13} , ω_{kmn}^{i14} , ω_{kmn}^{i15} , and ω_{kmn}^{i16} , auxiliary variables, γ_{kmn}^{i13} , γ_k^{i14} , γ_{kmn}^{i15} , and γ_k^{i16} , and constraints (B.53)-(B.65) correspond the original constraints (61).

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Fig. 1. DWPT Demonstration at USU.

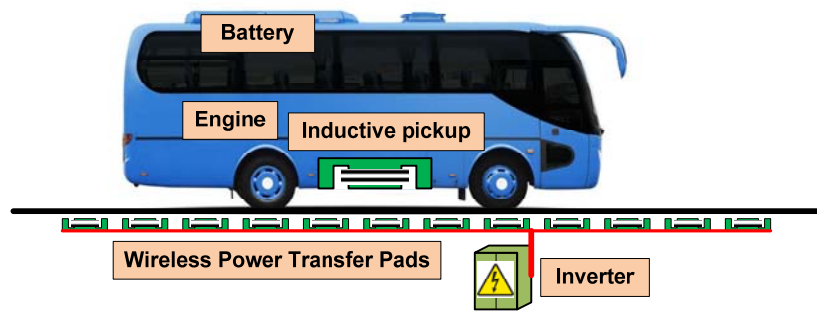


Fig. 2. A DWPT Facility.

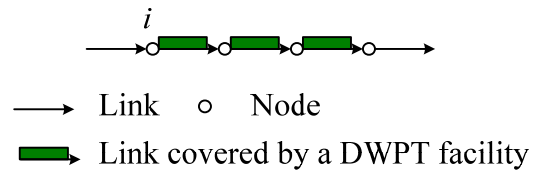


Fig. 3. An example of a starting point of a DWPT facility.

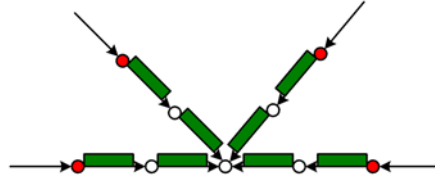
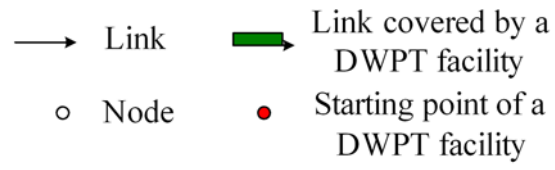


Fig. 4. A DWPT facility that covers an intersection.

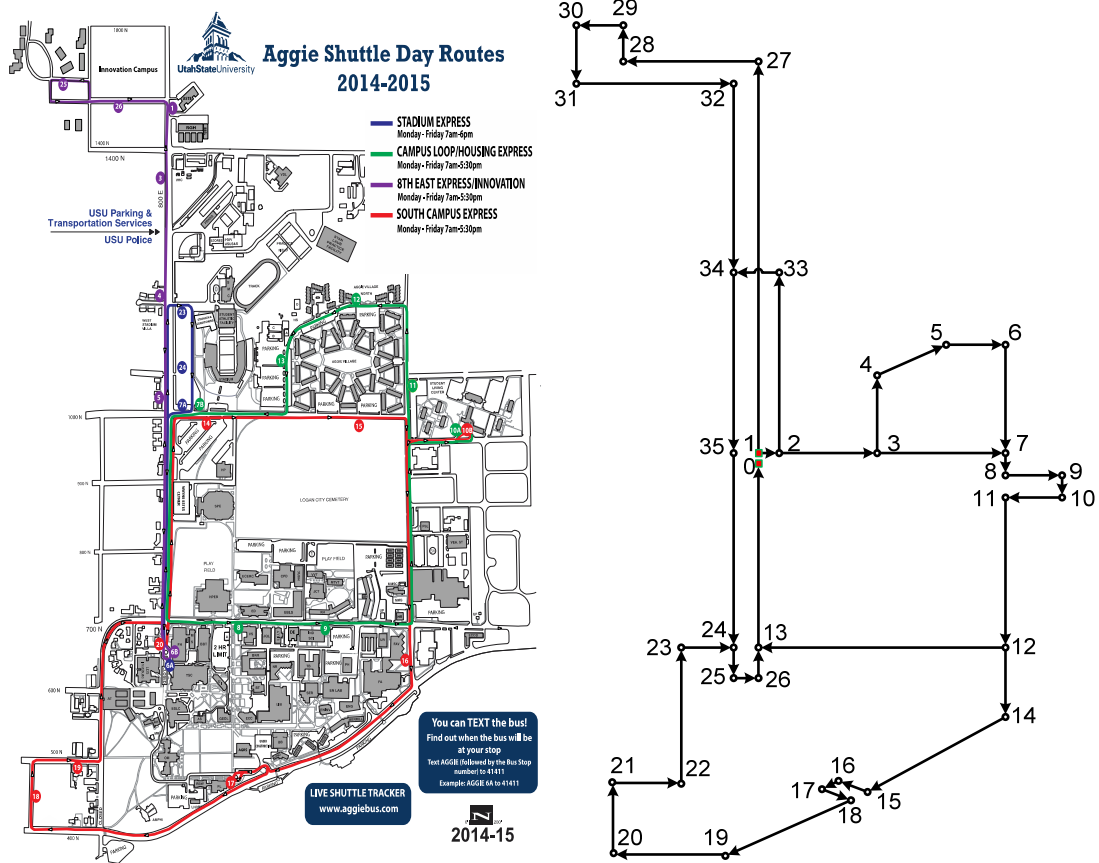


Fig. 5. USU campus bus system. (a) bus route map; (b) network representation.

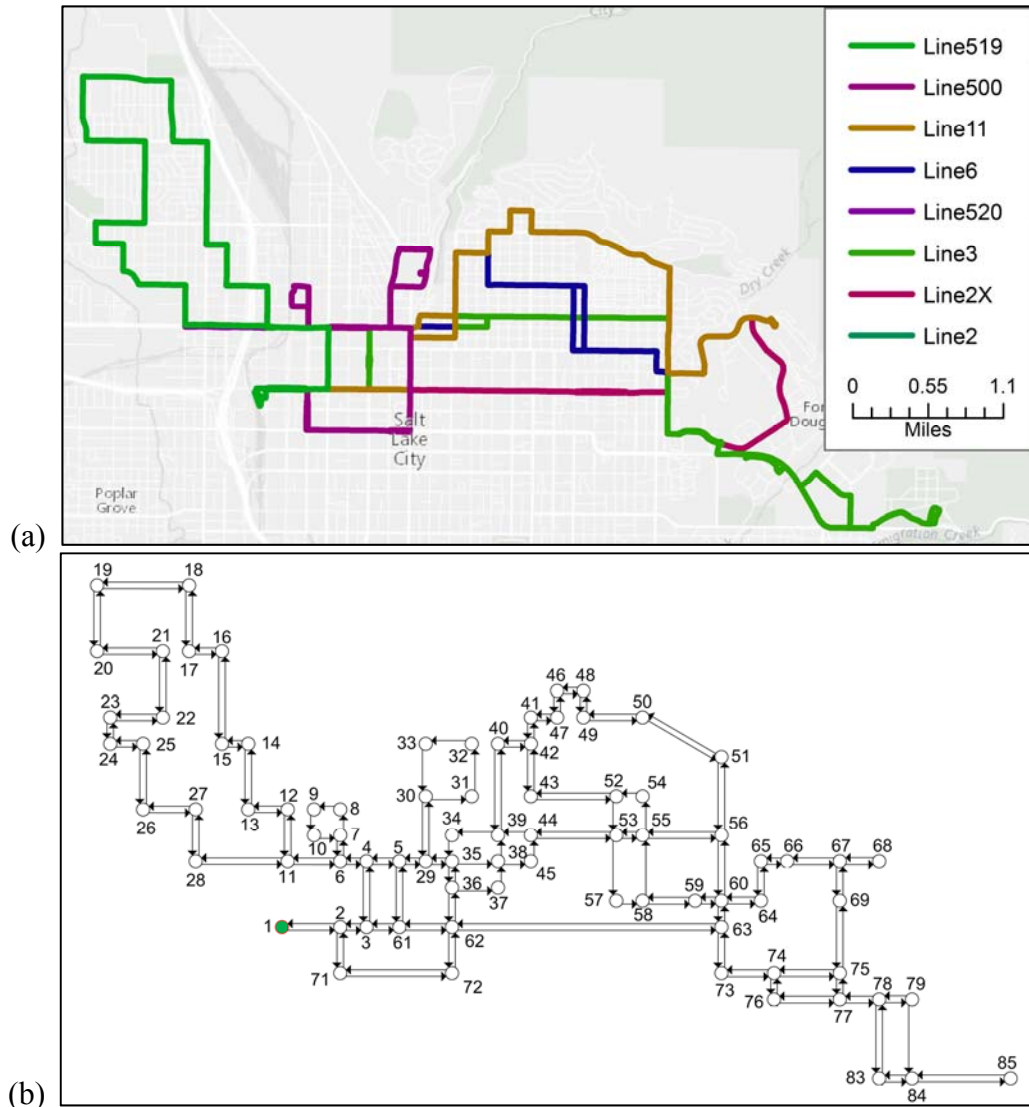


Fig. 6. SLC bus system. (a) bus route map; (b) network representation.

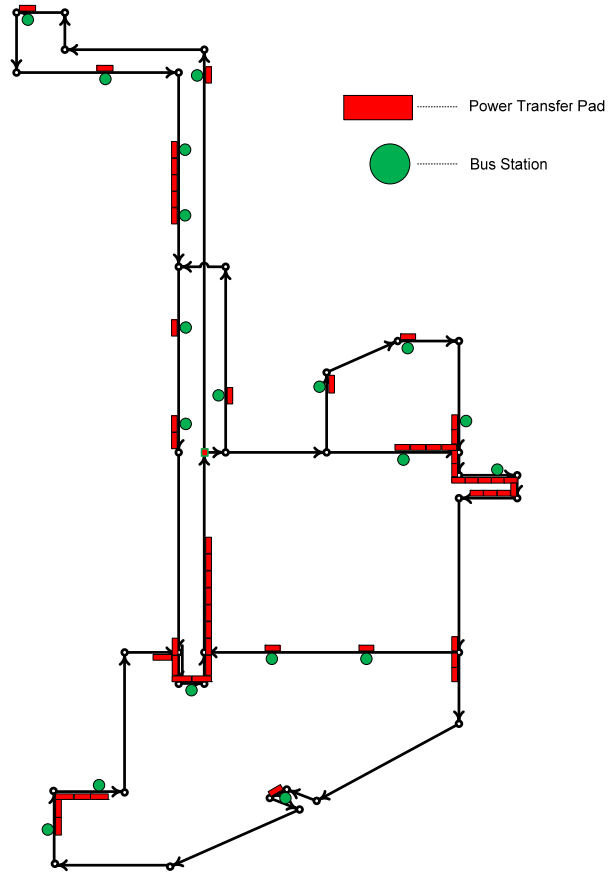


Fig. 7. The optimal layout of power transmitters.

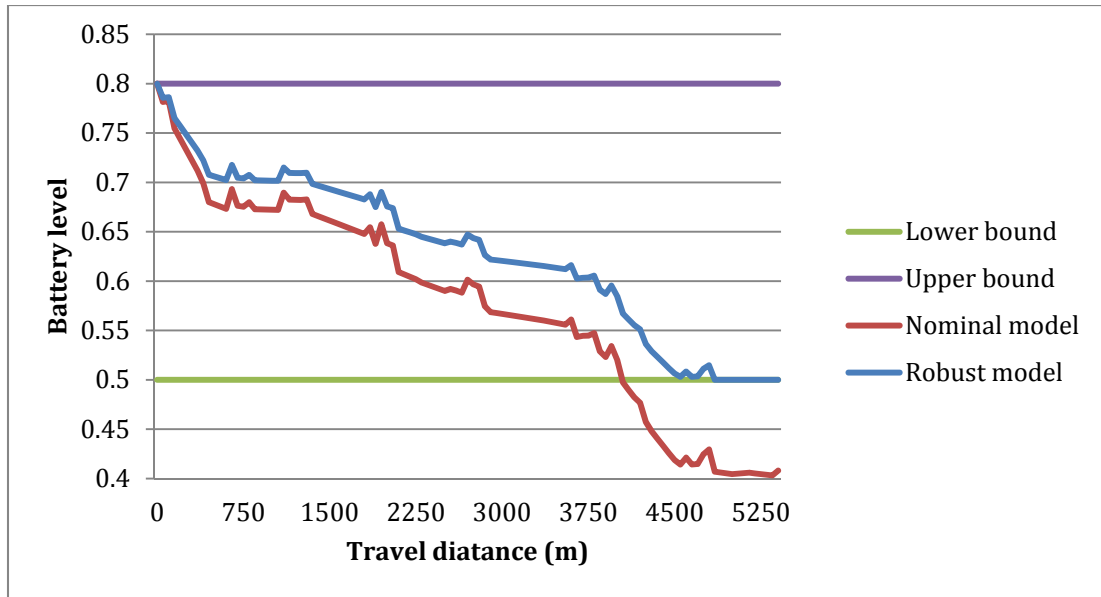


Fig. 8. Comparison of the battery level profile of the red (#1) bus line between the deterministic model solution and the robust model solution.

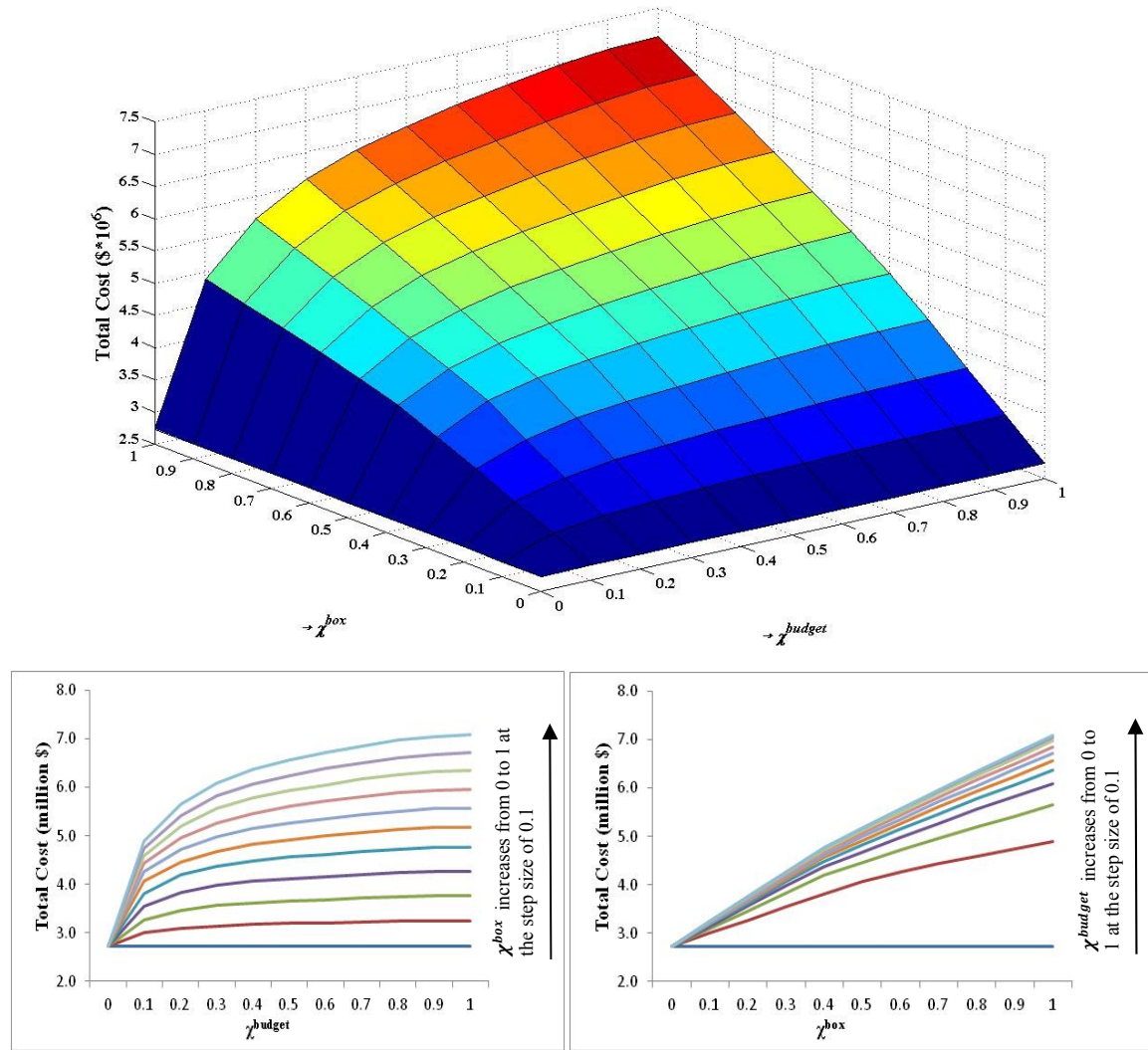


Fig. 9. The total cost of the DWPT electric bus system under different uncertainty levels.

Table 1

Decision variables about DWPT facilities.

Variables	Type	Domain of Definition	Description
x_{ij}	Binary	Set of all links L	$x_{ij} = 1$ is equivalent to that a DWPT facility covers link (i, j) .
y_i	Binary	Set of all nodes N	$y_i = 1$ is equivalent to that node i is a starting point of a DWPT facility.
z_i	Binary	Set of intersection nodes N^s	$z_i = 1$ is equivalent to that an incoming link (m, i) of node i is covered by a DWPT facility.

Table 2

Decision variables about batteries.

Variables	Type	Domain of Definition	Description
e_k^{max}	Real	Set of all electric bus lines	e_k^{max} represents the battery size for an electric bus line k .
e_{ki}	Real	Set of all nodes and all electric bus lines	e_{ki} represents the battery level at node i for an electric bus line k .

Table 3

Service loop of four lines.

Line	Loop	Number of buses
Red	1-2-3-7-8-9-10-11-12-14-15-16-17-18-19-20-21-22-23-24-25-26-13-0	4
Green	1-2-3-4-5-6-7-8-9-10-11-12-13-0	4
Blue	1-2-33-34-35-24-25-26-13-0	4
Purple	1-27-28-29-30-31-32-34-35-24-25-26-13-0	4

Table 4

Service Loops and Number of Buses for 8 Lines.

Line	Service Loop	Number of Buses
2	1-2-3-61-62-63-60-64-65-66-67-68-67-66-65-64-60-63-62-61-3-2-1	6
2X	1-2-3-61-62-63-73-74-75-69-67-68-67-66-65-64-60-63-62-61-3-2-1	6
3	1-2-3-61-5-29-35-38-45-44-53-55-56-60-63-73-74-76-77-78-79-84-85-84-83-78-77-76-74-73-63-60-56-55-53-44-39-34-35-29-5-61-3-2-1	4
6	1-2-3-4-5-29-35-38-39-40-42-43-52-53-57-58-59-60-64-65-66-67-68-67-66-65-64-60-59-58-55-54-52-43-42-40-39-34-35-29-5-4-6-7-8-9-10-7-6-4-3-2-1	4
11	1-2-3-61-62-36-37-38-39-40-42-41-47-46-48-49-50-51-56-60-64-65-66-67-68-67-66-65-64-60-56-51-50-49-48-46-47-41-42-40-39-34-35-36-62-61-3-2-1	4
500	1-2-71-72-62-36-35-29-30-31-32-33-30-29-5-4-6-7-8-9-10-7-6-4-5-29-35-36-62-72-71-2-1	4
519	1-2-3-4-6-11-28-27-26-25-24-23-22-21-20-19-18-17-16-15-14-13-12-11-6-4-3-2-1	3
520	1-2-3-4-6-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-11-6-4-3-2-1	3

Table 5

Parameters pertaining to energy consumption model.

Notation	Description	Value
ϖ_{ij}	Friction factor	0.02
w_{kij}^{fix}	Fixed part of total mass (kg)	20,400
ε	Gravity acceleration (m/s^2)	9.81
ρ	Air density (kg/m^3)	1.2
σ	Air resistance coefficient	0.7
Γ_k	Bus frontal area (m^2)	7.5
η_k^{out}	Energy output efficiency	60%
η_k^{in}	Energy input efficiency	50%
b	Weight of battery pack per unit capacity (kg/kWh)	11.36

Table 6

Parameters pertaining to DWPT facilities and batteries.

Notation	Description	Value
p	Energy supply rate (kW)	80
a^{fix}	Amortized fixed cost of power transmitters (\$)	20,000/30
a^{var}	Amortized variable cost of power transmitters (\$)	200/30
a^{bat}	Amortized cost of battery (\$/kWh)	100
ζ_k	Number of electric buses on line k	4
ϵ_k^{lo}	Battery level lower bound coefficient	0.5
ϵ_k^{up}	Battery level upper bound coefficient	0.8

Table 7

Results of the nominal model.

Result		Value
Total cost (30 years)		\$2,731,724
Battery size	Red (#1)	55.6 kWh
	Green (#2)	21.8 kWh
	Blue (#3)	32.5 kWh
	Purple (#4)	45.2 kWh
Total battery cost (30 years)		\$1,861,724
Number of DWPT facilities		16
Total fixed cost of DWPT facilities (30 years)		\$320,000
Total length of DWPT facilities		2,750 m
Total variable cost of DWPT facilities (30 years)		\$550,000

Table 8

Battery size comparison between the DWPT electric bus system and the stationary charging electric bus system.

Shuttle Line	Battery Capacity(kWh)		Battery Size Reduction
	Stationary Charging	DWPT	
Red (#1)	97.7	55.6	43.1%
Green (#2)	54.5	21.8	60.0%
Blue (#3)	47.5	32.5	31.6%
Purple (#4)	86.4	45.2	47.7%

Table 9

Total cost comparison between the DWPT electric bus system and the stationary charging electric bus system.

Items	Cost(\$)		
	Stationary charging	DWPT	
Battery	3,432,497	1,861,724	
Power track fixed cost	-	320,000	
Power track variable cost	-	550,000	Total cost reduction
Total	3,432,497	2,731,724	20.4%

Table 10Comparison between the deterministic model and the robust model ($\chi^{box} = 0.1, \chi^{budget} = 1.0$).

Result	Value	
	Deterministic model	Robust model
Total cost (30 years)	\$2,731,724	\$3,242,080
Battery size		
Red (#1)	55.6 kWh	70.3 kWh
Green (#2)	21.8 kWh	28.2 kWh
Blue (#3)	32.5 kWh	39.1 kWh
Purple (#4)	45.2 kWh	58.4 kWh
Total battery cost (30 years)	\$1,861,724	\$2,352,080
Number of power transmitters	16	15
Total fixed cost of Power transmitters (30 years)	\$320,000	\$300,000
Total length of power transmitters	2750 m	2950m
Total variable cost of power transmitters (30 years)	\$550,000	\$590,000

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